

# A Theory of Experience Effects\*

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## Abstract

How do financial crises and stock-market fluctuations affect investor behavior and the dynamics of financial markets in the long run? Recent evidence suggests that individuals overweight personal experiences of macroeconomic shocks when forming beliefs and making investment decisions. We propose a theoretical foundation for such experience-based learning and derive its dynamic implications in a simple OLG model. Risk averse agents invest in a risky and a risk-free asset. They form beliefs about the payoff of the risky asset based on the two key components of experience effects: (1) they overweight data observed during their lifetimes so far, and (2) they exhibit recency bias. In equilibrium, prices depend on past dividends, but only on those observed by the generations that are alive, and they are more sensitive to more recent dividends. Younger generations react more strongly to recent experiences than older generations, and hence have higher demand for the risky asset in good times, but lower demand in bad times. As a result, a crisis increases the average age of stock market participants, while booms have the opposite effect. The stronger the disagreement across generations (e.g., after a recent shock), the higher is the trade volume. We also show that, vice versa, the demographic composition of markets significantly influences the response to aggregate shocks. We generate empirical results on stock-market participation, stock-market investment, and trade volume from the *Survey of Consumer Finances*, merged with CRSP and historical data on stock-market performance, that are consistent with the model predictions.

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# 1 Introduction

Economists and policy-makers alike have long wrestled with the long-lasting effects of financial crises and other macroeconomic shocks. In the case of the Great Depression, [Friedman and Schwartz \(1963\)](#) argue that the experience of that time created a “mood of pessimism that for a long time affected markets.” In the case of the recent financial crisis, [Blanchard \(2012\)](#) maintains that “the crisis has left deep scars, which will affect both supply and demand for many years to come.” The notion that the experience of macro-economic shocks can leave an imprint on individuals’ attitudes and willingness to take risk in the long-run is consistent with growing empirical evidence on *experience effects*. For example, [Malmendier and Nagel \(2011\)](#) show that the stock-market experiences of individual investors predict their future willingness to invest in the stock market, and [Kaustia and Knüpfer \(2008\)](#) argue the same for IPO experiences.<sup>1</sup>

The theoretical foundations of long-run crisis effects are still debated. Prior research points to altered investment behavior during recessions, rather than experience effects, causing “hysteresis effects” ([Delong and Summers \(2012\)](#)), or argues that we need to revise our understanding of the stochastic processes governing the economy to explain scaled down investments after a crisis, such as the “disasterization approach” proposed by [Gabaix \(2011, 2012\)](#). The interpretation of [Friedman and Schwartz \(1963\)](#) goes in a different direction. Their notion is that the experience of an economic crisis induces pessimism and alters expectations about the future, as also pointed out by [Cogley and Sargent \(2008\)](#). In a similar vein, [Woodford \(2013\)](#) has argued that the stylized facts emerging from bubbles and crises require us to step away from the rational-expectations hypothesis. Confirming this notion, much of the evidence on experience effects pertains directly to beliefs, e.g., expectations of future stock market performance in the UBS/Gallup data ([Malmendier and Nagel \(2011\)](#)),

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<sup>1</sup> There is also evidence of experience effects in non-financial settings. For example, [Oreopoulos, von Wachter, and Heisz \(2012\)](#) show the long-term effects of graduating in a recession on labor market outcomes, and [Alesina and Fuchs-Schundeln \(2007\)](#) relate the personal experience of living in (communist) Eastern Germany to political attitudes post-reunification. See also [Giuliano and Spilimbergo \(2013\)](#), who relate the effects of growing up in a recession to redistribution preferences.

inflation expectations in the Michigan Survey of Consumers ([Malmendier and Nagel \(2016\)](#)), or expectations of future unemployment rates and the outlook for durable consumption, also in the Michigan Survey of Consumers ([Malmendier and Shen \(2015\)](#)).

In this paper, we propose the first formal theoretical framework that captures both of the main empirical features of experience effects: (1) over-weighting lifetime experiences and (2) recency bias. This theoretical approach builds closely on the psychology evidence on availability bias, initiated by [Tversky and Kahneman \(1974\)](#), and on the extensive evidence on the different effects of description versus experience.<sup>2</sup> Our framework is designed to study the long-term effects of personal experiences on the cross-section of stock-market participation and portfolio decisions, as well as on the time series of equilibrium prices, trade volume, and other financial market aggregates. It generates testable predictions about trading behavior and about the cross-sectional composition of stock-market investors, which relate to long-standing empirical puzzles such as the excess volatility puzzle ([LeRoy and Porter \(1981\)](#), [Shiller \(1981\)](#), [LeRoy \(2005\)](#)) or the predictive power of dividend-price ratios for future stock returns ([Campbell and Shiller \(1988\)](#)). We also take the model predictions to the data, and find evidence on the cross-section of stock-market participation, the cross-section of asset holdings, and trade volume that are consistent with our model. While more evidence on the exact process of household-level learning is needed to accompany the theoretical development (see the discussions in [Campbell \(2008\)](#) and [Agarwal, Driscoll, Gabaix, and Laibson \(2013\)](#)), our model aims to lay the foundation for testing whether experience-based learning can provide a new framework for expectation formation that would allow to capture the above mentioned stylized facts from macro-finance. It also allows us to explore the aggregate dynamics of an economy with experience-based learners.

We develop a stylized overlapping generations (OLG) general equilibrium model in which agents form their beliefs by overweighting their own experiences. Investors have CARA preferences and live for a finite number of periods. During their lifetimes, they choose portfolios

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<sup>2</sup> See, for example, [Weber, Böckenholt, Hilton, and Wallace \(1993\)](#), [Hertwig, Barron, Weber, and Erev \(2004\)](#), and [Simonsohn, Karlsson, Loewenstein, and Ariely \(2008\)](#).

of a risky and a risk-free security to maximize their per-period payoffs. The risky asset is in unit net supply and pays random dividends every period. The risk-less asset is in infinitely elastic supply and pays a fixed return. Investors do not know the true mean of the distribution of dividends, but they learn about it by observing the history of dividends.

The novel feature of the model is that investors are *experience-based learners*. That is, they over-weight the outcomes they have experienced in their lives when forming beliefs about the mean of dividends. Specifically, we assume that when forming their beliefs agents (i) only use data observed during their lifetimes, and (ii) may over-weight more recent observations. These two assumptions capture, in a simplified form, the psychology evidence on availability bias as first discussed by [Tversky and Kahneman \(1974\)](#). We contrast these assumptions with the standard model of Full Bayesian Learners (FBL) and also an alternative we dub “Bayesian Learning from Experience” (BLE). In a world of FBL agents use all available data and do not display recency bias. Their beliefs do not differ across cohorts and, eventually, will converge to the truth. In a world of BLE agents, i.e., where (i) holds but not (ii), this is not true. Cohorts differ in their beliefs. However, BLE does not allow for the empirically documented recency bias. As such BLE is akin to over-extrapolation in the spirit of [Barberis et al. \(2015\)](#) and [Barberis et al. \(2016\)](#), but applied to individuals’ cohort-specific lifetimes rather than to some cohort-independent number of recent periods.

Our stylized model allows us to fully isolate the forces introduced by the presence of experience-based learners. We begin by characterizing the benchmark economy in which agents know the true mean of dividends. In this setting, the model features constant equilibrium prices, since agent’s demands and the asset supply are constant over time. The cross section of equilibrium asset holdings is constant over time due to the lack of disagreements among agents. Any departure from this benchmark can thus be cleanly attributed to experience-based learning.

We then introduce experience-based learning into the model, and identify long-lasting effects of economic shocks on equilibrium prices, trade volume, and the cross section of asset

holdings. We emphasize two channels. The first channel is the belief-formation process: Shocks to dividends shape agents' beliefs about future dividends. Each cohort uses the dividends observed during their lifetimes so far to form their beliefs. Thus, the aggregate demand for the risky asset depends on the weighted sum of cohorts' beliefs about the payoff of this asset. As a result, the market-clearing price is a function of the history of dividends observed by at least one market participant.

The second channel through which experience-based learning matters for equilibrium outcomes is the generation of cross-sectional heterogeneity in the population. Different lifetime experiences generate persistent belief heterogeneity among cohorts; agents in this model “agree to disagree.” Furthermore, given a common experience, different cohorts react differently to the same macroeconomic shock. Younger cohorts react more strongly than older cohorts as this new experience makes up a larger part of their lifetimes so far. As a result, a positive shock induces younger cohorts to invest relatively more in the risky asset, while a negative shock tilts the composition towards older cohorts. In fact, we show that periods of booms, interpreted as periods with sustained increases in dividends, result in younger generations holding a larger share of the risky asset than older generations, and vice-versa.

Relative to the benchmark of agents knowing the distribution of dividends, experienced-based learning introduces excess volatility, auto-correlation of prices, as well as return predictability. The extent of these features goes above and beyond the stochastic structure of the assumed dividend process. In addition, the model also generates implications for the time series of trade volume. We show that changes in the level of disagreement between cohorts lead to higher trade volume in equilibrium. The mechanism is intuitive: an increase (decrease) in dividends induces trade since young agents become more optimistic (pessimistic) than old agents, and disagreement generates gains from trade.

The model captures an interesting tension between heterogeneity in personal experiences (which generates belief heterogeneity across cohorts), and recency bias (which reduces belief heterogeneity): When there is strong recency bias, all agents pay a lot of attention to the

most recent dividend realization and, thus, their reactions to a given recent shock are similar. This increases the aggregate response to a shock and reduces heterogeneity across cohorts. As a result, price volatility increases and price auto-correlation and trade volume decrease. The opposite holds when the recency bias is weak, and agents form their beliefs using their experienced history.

We further explore the connection between demographics and the long-lasting effects of macroeconomic shocks by analyzing the effect of a one-time demographic shock to the economy. We find that the demographic composition of markets significantly influences the response to aggregate shocks. For example, when a demographic change increases the stock-market participation of the young relative to the old, the reliance of prices on most recent dividends relative to past dividends also increases. The demographic predictions are in line with evidence in [Cassella and Gulen \(2015\)](#), who estimate how much investors' recent return experiences, relative to older return experiences, help predict their expectations about future returns. They find a positive relation between this market-wide measure of experience (which they dub extrapolation bias) and the relative participation of young versus old investors in the stock market.

Our theoretical predictions are also consistent with a number of empirical results on portfolio decisions and trade volume. Using the representative sample of the *Survey of Consumer Finance*, merged with CRSP and historical data on stock-market performance, we show that cross-cohort differences in lifetime stock-market experiences predict cohort differences both in their stock-market participation and in the fraction of their liquid assets that they invest in the stock market. In other words, the cross-cohort differences both on the extensive and on the intensive margin of stock market participation vary over time as predicted by the time series of cross-cohort differences in lifetime experiences. We also show that, in terms of abnormal trade volume, the de-trended turnover ratio is strongly correlated with differences in lifetime experiences of stock-market returns across cohorts.

As the final step in our analysis, we investigate to what extent our results might be driven

by the (stylized) myopic formulation of our model. We extend the model of experience-based learning to a dynamic portfolio set-up where agents re-balance their portfolios every period to maximize their final-period consumption.<sup>3</sup> The dynamic set-up allows us to analyze how hedging concerns and lifetime-horizon effects interact with experience-based learning. Prior literature has shown that, in a rational expectations linear equilibrium, the agents' multi-period investment problem can be partitioned into a sequence of one-period ones (Vives (2010)). Under experience-based learning, such partitioning is no longer possible. Future beliefs and portfolio decisions of experience-based learners and, as a result, future prices depend on current dividends, making investors' wealth in the distant future correlated with next period's returns. By exploiting the CARA-Gaussian setup, however, we are able to show that the demand of experience-based learners coincides with the one in a static problem where dividends are drawn from a *modified* Gaussian distribution. That is, we can still partition the multi-period investment problem into a sequence of one-period problems, albeit with a probability distribution of dividends that differs from the original one. This latter result might also be of interest as an independent technical contribution in solving belief dependencies beyond the specific model proposed in this paper.

In this dynamic portfolio problem, we decompose agents' demands for the risky asset into three demand motives: a belief term, a hedging term, and a horizon term. The *belief term* is given by the demand of the myopic agents in our baseline model above. All the forces that are present in the baseline model are captured by this term. The two additional terms capture the dynamics inherent to the multi-period problem: The *hedging term* captures that agents anticipate that they will learn about the risky asset from future dividends, and that this future learning will in turn affect prices and future returns. In order to hedge their exposure to changes in beliefs, they distort their portfolio decisions relative to the static model. The *horizon term* captures that younger agents react less aggressively to a given change in beliefs due to their longer remaining investment horizon. Thus, their longer investment horizon

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<sup>3</sup> This form of modeling of dynamic portfolio choices is standard in the literature, see Vives (2010).

makes them behave in a more risk-averse fashion. We focus on a two-period setting and show that the qualitative results presented in the baseline model, where agents maximize their per-period utility, pass through to the dynamic portfolio problem.

In summary, our paper provides a simple formalization of experience effects. It generates testable implications for individual financial decision-making and the resulting stock-market dynamics, including the long-term effects of crisis experiences. The model, together with our empirical findings, suggest that a deeper understanding of the influence of past experiences is important not only to improve the micro-modeling of financial risk-taking, but also for our understanding of the aggregate dynamics of financial markets and the long-run effects of macro-shocks.

**Related Literature.** The above-cited empirical literature on experience effects shows that personal experiences of macroeconomic shocks leave a lasting imprint on individuals’ decision-making over their lifetimes, thus generating long-run effects of macroeconomic crises. Our paper provides a theoretical foundation for such behavior. Closely related to our approach, [Cogley and Sargent \(2008\)](#) propose a model in which the representative consumer uses Bayes’ theorem to update estimates of transition probabilities as realizations accrue. As in our paper, agents use less data than in a standard framework (i.e., less “than a rational-expectations-without-learning econometrician would give them,” as the authors put it). There are two important differences in our setup. First, agents in our model are not Bayesian due to the presence of availability bias and recency bias in their belief formation process. Second, and most importantly, we model over-lapping generations to capture the fact that agents live for a finite number of periods and that different cohorts have different experiences. Consequently, observations during an agents’ lifetime have a non-negligible effect on their beliefs and generate heterogeneity across market participants. This feature provides an alternative modeling device to capture Friedman and Schwartz’s idea that economic events, such as the Great Depression, shape the attitude of agents towards financial markets in the future.

Our paper also relates to the work on extrapolation by [Barberis et al. \(2015\)](#) and [Bar-](#)



beris et al. (2016). Their work also departs from the Bayesian paradigm by considering a consumption-based asset pricing model populated by “rational” agents and “extrapolative” agents. Extrapolative agents believe that positive changes in prices will be followed by positive changes. One main difference to our paper is the approach to modeling agents’ beliefs. In our model, agents hold misspecified beliefs over (the mean of) future dividends, but hold correct beliefs of the mapping between equilibrium prices and dividends. A second, and perhaps more important, difference relates to the sources of heterogeneity in each model. In their framework of infinitely lived agents, cross-sectional heterogeneity arises due the presence of both “rational” and “extrapolative” agents. In our model, heterogeneity results from different cohorts of finitely-lived agents assigning different weights to past dividends when forming their beliefs. Each cohort’s beliefs are the result of the cohort’s lifetime experiences so far. The latter features allows us to study the link between personal experiences and demographic structure on equilibrium outcomes.

More generally, our paper relates to a large literature in asset pricing that departs from the correct-beliefs paradigm. For instance, Barsky and DeLong (1993), Timmermann (1993), Timmermann (1996), and Adam, Marcet, and Nicolini (2012) study the implications of learning for stock-return volatility and predictability. Cecchetti, Lam, and Mark (2000) construct a Lucas asset-pricing model with infinitely-lived agents where the representative agent’s subjective beliefs about endowment growth are distorted. On a similar note, Jin (2015) rationalizes financial booms and busts in a model where agents learn about the probability of a crash, but hold incorrect beliefs about the underlying process of this risk.

At the same time, our approach is different from asset pricing models with asymmetric information, as surveyed in Brunnermeier (2001). A key distinction between experienced-based learning and models where agents have private information is that, in the former, information is available to all agents, while in the latter agents want to learn the information their counter-parties hold. Experience-based learners choose to down-weight the observations they have not directly observed when forming their beliefs, even though such observations

are available to them (and to all other agents).

Finally, there are contemporaneous papers to ours exploring the role of learning in overlapping generations models (Collin-Dufresne, Johannes, and Lochstoer (2014), Schraeder (2015)). The paper most closely related to ours is Ehling, Graniero, and Heyerdahl-Larsen (2015), who explore the role of experience in portfolio decisions and asset prices in a complete markets setting. Differently from our paper, they do not aim to capture “experience effects” in the sense of the empirically observed pattern in Malmendier and Nagel (2011), which involves a declining weighting function and thus recency bias. Instead, they are interested in the pure effect of individuals restricting their use of data to their lifetimes. Similar to the *Bayesian Learners from Experience* in our analysis, agents in their paper start from a given prior (the truth) which they update only using lifetime observations. The authors use this setting to develop a theoretical underpinning for trend chasing and the negative relationship between beliefs about expected returns and realized future returns, as shown by Greenwood and Shleifer (2014). Instead, our incomplete markets setting allows us to focus on the cross-section of asset holdings and the relation between trade volume and price behavior in the presence of recency bias.

There is a large literature which proposes other mechanisms, such as borrowing constraints or life-cycle considerations, as the link from demographics to asset prices and other equilibrium quantities. We view these other mechanisms as complementary to our paper. They are omitted for the sake of tractability of the model.

The remainder of the paper is organized as follows. In Section 2, we present the model setup and describe the notion of experience-based learning. In Section 3, we illustrate the mechanics of the model and main results in a simplified version of our model. We present our main results in Section 4, and we extend the model to study demographic shocks in Section 5. In Section 6, we present stylized facts that are in line with our model predictions, and we extend the model to non-myopic agents in Section 7. We conclude in Section 8.

## 2 Model Set-Up

### 2.1 Lucas-Tree Economy

Consider an infinite-horizon economy with overlapping generations of a continuum of risk-averse agents. At each  $t \in \mathbb{Z}$ , a new generation is born and lives for  $q$  periods, with  $q \in \{1, 2, 3, \dots\}$ . Hence, there are  $q + 1$  generations alive at any  $t$ . The generation born at time  $t = n$  is called generation  $n$ . Each generation has a mass of  $q^{-1}$  identical agents.

Agents have CARA preferences with risk aversion  $\gamma$ . They are born with no endowment and can transfer resources across time by investing in financial markets. Trading takes place at the beginning of each period. At the end of the last period of their lives, agents consume the wealth they have accumulated. We use  $n_q$  to indicate the last time at which generation  $n$  trades,  $n_q = n + q - 1$ . (If the generation is denoted by  $t$  we use  $t_q$ .) Figure 1 illustrates the timeline of this economy for two-period lived generations ( $q = 2$ ).

There is a risk-less asset, which is in perfectly elastic supply and has a (gross) payoff of  $R > 1$  at all times. There is a single risky asset (a Lucas Tree), which is in unit net supply and pays a random dividend  $d_t \sim N(\theta, \sigma^2)$  at time  $t$ . To model uncertainty about fundamentals, we assume that agents do not know the true mean of dividends  $\theta$  and use past observations to estimate the mean. To keep the model tractable, we assume that the variance of dividends  $\sigma^2$  is known at all times.

For each generation  $n \in \mathbb{Z}$  and any  $t \in \{n, \dots, n + q\}$ , the budget constraint is given by

$$W_t^n = x_t^n p_t + a_t^n, \quad (1)$$

where  $W_t^n$  denotes the wealth of generation  $n$  at time  $t$ ,  $x_t^n$  is the investment in the risky asset (units of Lucas Tree output),  $a_t^n$  is the amount invested in the riskless asset, and  $p_t$  is the price of one unit of the risky asset at time  $t$ . As a result, wealth next period is

$$W_{t+1}^n = x_t^n (p_{t+1} + d_{t+1}) + a_t^n R = x_t^n (p_{t+1} + d_{t+1} - p_t R) + W_t^n R. \quad (2)$$

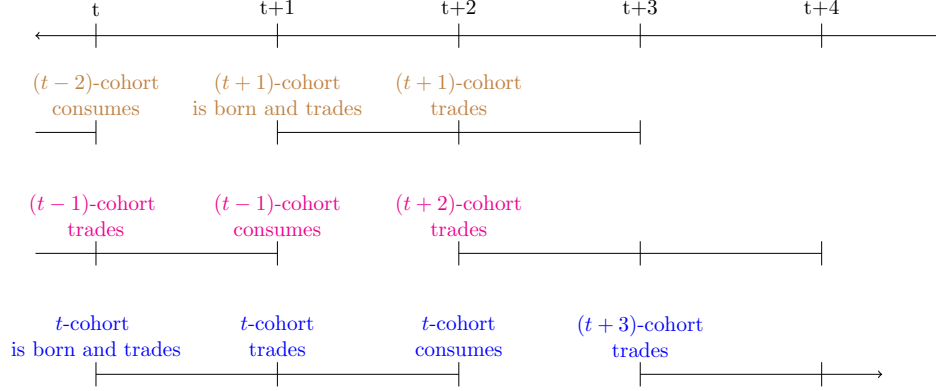


Figure 1: A timeline for an economy with two-period lived generations,  $q = 2$ .

We denote the net (or excess) payoff received in  $t+1$  from investing in one unit of the risky asset at time  $t$  as  $s_{t+1} \equiv p_{t+1} + d_{t+1} - p_t R$ . Note that  $p_{t+1} + d_{t+1}$  is the payoff received at  $t+1$  from investing in one unit of the risky asset at time  $t$ , and  $p_t R$  is the (opportunity) cost of investing in one unit of the risky asset at time  $t$ . Using this notation,  $W_{t+1}^n = x_t^n s_{t+1} + W_t^n R$ .

In the baseline version of our model, we assume that agents are myopic and maximize their per-period utility. This assumption simplifies the maximization problem considerably, and highlights the main determinant of portfolio choice generated by experience-based learning. (In Section 7, we remove this assumption and show that the same mechanism is at work and shapes the results.)

For a given initial wealth level  $W_n^n$ , the myopic problem of a generation  $n$  at each time  $t \in \{n, \dots, n_q\}$  is to choose  $x_t^n$  such that it solves  $\max_{x \in \mathbb{R}} E_t^n [-\exp(-\gamma W_{t+1}^n)]$ , and hence

$$x_t^n \in \arg \max_{x \in \mathbb{R}} E_t^n [-\exp(-\gamma x s_{t+1})]. \quad (3)$$

Given that agents only need to learn about the mean of dividends,  $E_t^n [\cdot]$  is the (subjective) expectation with respect to a Gaussian distribution with variance  $\sigma^2$  and a mean denoted by  $\theta_t^n$ . We call  $\theta_t^n$  the subjective mean of dividends, and we will define it below. Note that, when  $x_t^n$  is negative, generation  $n$  is short-selling units of the Lucas tree.

## 2.2 Experience-Based Learning

In this framework, experienced-based learning (EBL) means that agents overweight observations received during their lifetimes when forecasting dividends, and that they tilt the excess weights towards the most recent observations. For simplicity, we assume that agents *only* use observations realized during their lifetimes.<sup>4</sup> That is, even though they observe the entire history of dividends, EBL agents choose to disregard observations outside their lifetimes. Note that, in this full-information setting, prices do not add any additional information. While it is possible to add private information and learning from prices to our framework, these (realistic) feature would complicate matters without necessarily adding new intuition.

EBL differs from reinforcement learning-type models in two ways. First, as already discussed, EBL agents understand the model and know all the primitives, except the mean of the dividend process. Hence, they do not learn *about* the equilibrium, they learn *in* equilibrium. Second, EBL features a passive learning problem in the sense that actions of the players do not affect the information they receive. This would be different if we had, say, a participation decision that would link an action (participate or not) to the type of data obtained for learning. We consider this to be an interesting line to explore in the future.

We construct the subjective mean of dividends of generation  $n$  at time  $t$  following Malmendier and Nagel (2010):

$$\theta_t^n \equiv \sum_{k=0}^{age} w(k, \lambda, age) d_{t-k}, \quad (4)$$

where  $age = t - n$ , and where, for all  $k \leq age$ ,

$$w(k, \lambda, age) = \frac{(age + 1 - k)^\lambda}{\sum_{k'=0}^{age} (age + 1 - k')^\lambda} \quad (5)$$

denotes the weight an agent aged  $age$  assigns to the dividend observed  $k$  periods earlier, and  $w(k, \lambda, age) \equiv 0$  for all  $k > age$ . That is, agents put weight  $\frac{(age+1)^\lambda}{\sum_{k'=0}^{age} (age+1-k')^\lambda}$  on the most

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<sup>4</sup> All we need for our results to hold is that agents discount their pre-lifetime history relative to their experienced history when forming beliefs.

recent observation,  $\frac{(age+1-1)^\lambda}{\sum_{k'=0}^{age} (age+1-k')^\lambda}$  on the previous one, and so forth for all observations experienced during their lifetimes so far, and no weight on earlier observations. The sum of all weights an agent applies to her lifetime experiences so far is always equal to one,  $\sum_{k=0}^{age} w(k, \lambda, age) = 1$  for all  $age \in \{0, 1, \dots, q\}$ . For example, if  $q = 2$ , the older generation, which that has lived for one period, uses weights  $w(0, \lambda, 1) = \frac{2^\lambda}{1+2^\lambda}$  on the current realization  $d_t$ , and  $w(1, \lambda, 1) = \frac{1}{1+2^\lambda}$  on the previous one,  $d_{t-1}$ , while the younger generation born in  $t$  places full weight,  $w(0, \lambda, 0) = 1$ , on the current observation  $d_t$ .

The denominator in (5) is a normalizing constant that depends only on  $age$  and  $\lambda$ . The parameter  $\lambda$  regulates the relative weights of earlier and later observations. For  $\lambda > 0$ , more recent observations receive relatively more weight, whereas for  $\lambda < 0$  the opposite holds. Here are three examples of possible weighting schemes:

**Example 2.1** (Linearly Declining Weights,  $\lambda = 1$ ). *For  $\lambda = 1$ , weights decay linearly as the time lag increases, i.e., for any  $0 \leq k, k + j \leq age$ ,*

$$w(k + j, 1, age) - w(k, 1, age) = -\frac{j}{\sum_{k'=0}^{age} (age + 1 - k')} = -\frac{2j}{(age + 1)(age + 2)}.$$

**Example 2.2** (Equal Weights,  $\lambda = 0$ ). *For  $\lambda = 0$ , lifetime observations are equal-weighted, i.e., for any  $0 \leq k \leq age$ ,*

$$w(k, 0, age) = \frac{1}{age + 1}.$$

**Example 2.3.** *For  $\lambda \rightarrow \infty$ , the weight assigned to the most recent observation converges to 1, and all other weights converge to 0, i.e., for any  $0 \leq k \leq age$*

$$w(k, \lambda, age) \rightarrow 1_{\{k=0\}}.$$

Observe that by construction,  $\theta_t^n \sim N(\theta, \sigma^2 \sum_{k=0}^{age} (w(k, \lambda, age))^2)$ . Hence,  $\theta_t^n$  does not necessarily converge to the truth as  $t \rightarrow \infty$ ; it depends on whether  $\sum_{k=0}^{age} (w(k, \lambda, age))^2 \rightarrow 0$ .

This in turn depends how fast the weights for “old” observations decay to zero (i.e., how small  $\lambda$  is). When agents have finite lives, convergence will not occur. In addition, since separate cohorts weight different realizations differently, we should expect belief heterogeneity, driven by different experiences, at any point in time.

We conclude this section by showing a useful property of the weights, which is used in the characterization results below.

**Lemma 2.1.** *[Single-Crossing Property] Let  $age' < age$  and  $\lambda > 0$ . Then the function  $w(\cdot, \lambda, age) - w(\cdot, \lambda, age')$  changes signs (from negative to positive) exactly once over  $\{0, \dots, age' + 1\}$ .<sup>5</sup>*

*Proof.* See Appendix A. □

### 2.3 Comparison to Bayesian Learning

To better understand the experience-effect mechanism, we compare EBL agents to agents who update their beliefs using Bayes rule. We consider two cases: the standard case of Bayesian learning, wherein agents use all the available observations to form their beliefs; and an alternative case where agents only use data realized during their lifetimes, but update their beliefs using Bayes rule. We call the former case *Full Bayesian Learning* (FBL), and in the latter case *Bayesian Learning from Experience* (BLE).

**Full Bayesian Learners.** Full Bayesian learners use all the available observations “since the beginning of time” to form their beliefs. Formally speaking, there is no “beginning of time” in our economy since we are analyzing an economy that has been running forever. Hence, loosely speaking, the beliefs of Bayesian agents who use all available information would have converged to  $\theta$  at any point in time  $t$ , i.e., Bayesian agents would behave like agents that know the true mean. Here, however, we aim to illustrate the comparison of agents learning from experience to Bayesian agents learning from a common sample. Hence, we start the

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<sup>5</sup> Remember that we set  $w(k, \lambda, age) \equiv 0$  for all  $k > age$ .

economy at an initial time  $t = 0$  (for this analysis), and assume that all generations of FBL agents consider all observations since time 0 to form their belief. We denote the prior of FBL agents as  $N(m, \tau^2)$ . For simplicity, all generations have the same prior, though the analysis can easily be extended to heterogeneous Gaussian priors across generations.<sup>6</sup>

The posterior mean of any generation alive at time  $t$ ,  $\gamma_t$ , is given by

$$\gamma_t = \frac{\tau^{-2}}{\tau^{-2} + \sigma^{-2}t}m + \frac{\sigma^{-2}t}{\tau^{-2} + \sigma^{-2}t} \left( \frac{1}{t} \sum_{k=0}^t d_k \right).$$

That is, the belief of an FBL agent is a convex combination of the prior  $m$  and the average of all observations  $d_k$  available since time 0. The key difference to EBL agents is that, for FBL agents, different past experiences do not play a role, and hence there is no heterogeneity in posterior beliefs. All generations alive in any given period have the same belief about the mean of dividends. In addition, beliefs of FBL agents are non-stationary (i.e., depend on the time period), and as  $t \rightarrow \infty$ , the posterior mean converges (almost surely) to the true mean. That is, with FBL the implications of learning vanish as time goes to infinity. With EBL, this is not true. Since agents have finite lives and learn from their own experiences, our model generates learning dynamics even as time diverges.

**Bayesian Learners from Experience.** For BLE agents, the situation is different. We assume again that each generation has a prior  $N(m, \tau^2)$  when they are born. Here, the posterior mean of generation  $n$  at period  $t = n + \text{age}$ ,  $\beta_t^n$ , is given by

$$\beta_t^n = \frac{\tau^{-2}}{\tau^{-2} + \sigma^{-2}(\text{age} + 1)}m + \frac{(\text{age} + 1)\sigma^{-2}}{\tau^{-2} + \sigma^{-2}(\text{age} + 1)} \left( \frac{1}{\text{age} + 1} \sum_{k=n}^t d_k \right).$$

That is, the belief of a BLE generation is a convex combination of the prior  $m$  and the average of (only) the lifetime observations  $d_k$  available to date. In turn, this average coincides with the belief of our learners from experience  $\theta_t^n$  with  $\lambda = 0$ . Thus, the posterior mean of BLE agents and the belief of EBL agents with  $\lambda = 0$  differ in the weight on the prior mean and

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<sup>6</sup> The assumption of Gaussianity is also not needed but simplifies the exposition greatly.



in EBL agents not employing a prior. If the prior of BLE agents is diffuse, i.e.,  $\tau \rightarrow \infty$ , then  $\beta_{n+a}^n$  coincides with the  $\theta_{n+a}^n$  of EBL agents for  $\lambda = 0$ . Additionally, as *age* increases,  $\beta_{n+a}^n$  gets closer to  $\theta_{n+a}^n$ . For lower values of *age*, however, the presence of the prior will introduce a wedge between the BLE and EBL.

These two benchmark comparisons to Bayesian learning illustrate the role of experience-based learning, and in particular of the assumption that agents only use data observed during their lifetimes, in generating heterogeneity in beliefs. Under FBL, beliefs do not differ across agents and, eventually, will converge to the truth. Under BLE and our main approach, EBL, this is not true. Cohorts differ in their beliefs. However, BLE does not allow for the empirically documented recency bias. As such BLE is akin to over-extrapolation from one's lifetime.

An important difference between EBL, on the one hand, and both types of Bayesian agents, on the other hand, is that we assume that our EBL agents do not understand that their estimate for the mean of dividends is a random variable. A Bayesian agent would acknowledge that the perceived mean of dividends is random and, hence, her expected payoff will consist of two expectations – one with respect to dividends and one with respect to  $\theta$ . An EBL agent only has the former expectation and not the latter. That is, we assume that EBL agents, who form their beliefs about the mean as described in equation (4), make decisions as if this was the true mean of the dividends.

## 2.4 Equilibrium Definition

We now proceed to define the equilibrium of the economy with EBL agents.

**Definition 2.1** (Equilibrium). *An equilibrium is a demand profile for the risky asset  $\{x_t^n\}$ , a demand profile for the riskless asset  $\{a_t^n\}$ , and a price schedule  $\{p_t\}$  such that:*

1. *given the price schedule,  $\{(a_t^n, x_t^n) : t \in \{n, \dots, n_q\}\}$  solve the generation- $n$  problem, and*

2. the market clears in all periods, i.e.,

$$1 = \frac{1}{q} \sum_{n=t-q+1}^t x_t^n a \quad \forall t \in \mathbb{Z}. \quad (6)$$

We will focus the analysis on the class of linear equilibria, i.e., equilibria with affine prices:

**Definition 2.2** (Linear Equilibrium). *A linear equilibrium is an equilibrium wherein prices are an affine function of dividends. That is, there exists a  $K \in \mathbb{N}$ ,  $\alpha \in \mathbb{R}$ , and  $\beta_k \in \mathbb{R}$  for all  $k \in \{0, \dots, K\}$  such that:*

$$p_t = \alpha + \sum_{k=0}^K \beta_k d_{t-k}. \quad (7)$$

Thus, the price  $p_t$  is a linear function of the current and the last  $K$  dividends.

**Benchmark with known mean of dividends.** For the sake of benchmarking our results for EBL agents, we characterize the equilibrium of the economy where the mean of dividends,  $\theta$ , is known by all agents, i.e.,  $E_t^n(d_t) = \theta \forall n, t$ . In this scenario, there are no disagreements across cohorts, and the demand of any cohort trading at time  $t$  is given by:

$$x_t^n \in \arg \max_{x \in \mathbb{R}} E[-\exp(-\gamma x s_{t+1})], \quad (8)$$

i.e., substituting the subjective expectation  $E_t^n[\cdot]$  in (3) with the objective expectation  $E$ .

The solution to this problem is standard, and given by

$$x_t^n = \frac{E[s_{t+1}]}{\gamma V[s_{t+1}]} \quad (9)$$

for all  $n \in \{t-q+1, \dots, t\}$ , and zero otherwise. To derive this solution, we first guess that the price is constant,  $p_t = P \forall t$ . We verify this guess with the market clearing condition (6), and obtain  $P = \frac{\theta - \gamma \sigma^2}{R-1}$ . Furthermore, there is no heterogeneity in cohorts' portfolios, and thus, in equilibrium,  $x_t^n = 1$  for all  $n \in \{t-q+1, \dots, t\}$ , and zero otherwise.

### 3 Illustration: A Toy Model

To illustrate the mechanics of the model and to highlight, in a simple environment, the main results of the paper, we first solve the model for  $q = 2$ . We then generalize the model to any  $q > 1$  in the next section. (And we solve the non-myopic case in Section 7.)

In our toy model (i.e., for  $q = 2$ ), there are three cohorts alive at each point in time: a young cohort, which enters the market for the first time; a middle-aged cohort, which is participating in the market for the second time; and an old cohort, whose agents simply consume the payoffs from their lifetime investments. Since the old cohort has no impact on equilibrium prices or quantities, we focus our analysis on the behavior of the young and middle-aged agents. At time  $t$ , the problem of generations  $n \in \{t, t - 1\}$  is given by (3), and it is easy to show that their demands for the risky asset are thus given by

$$x_t^n = \frac{E_t^n[s_{t+1}]}{\gamma V_t^n[s_{t+1}]}.$$

As one of our first key results in Section 4, we will show that prices depend on the history of dividends and that this return predictability is limited to those past dividends that have been observed (experienced) by the oldest generation trading in the market. In other words, we will show that  $K = q - 1$  in equation (7). Anticipating this result here for the case  $q = 2$ , we have  $K = 1$  and thus

$$p_t = \alpha + \beta_0 d_t + \beta_1 d_{t-1}.$$

The dependence of prices on past dividends is an important feature of our model, which is shared by many models of extrapolation and learning. A distinct feature of our model, however, is that the dividends lag is intrinsically linked to the demographic structure and cross-sectional heterogeneity in lifetime experiences.

The cross-sectional differences in lifetime experiences, and the resulting cross-sectional differences in beliefs, determine cohorts' trading behavior. Given the functional form for

prices, we can re-write the demands of both cohorts that are actively trading as

$$x_t^t = \frac{\alpha + (1 + \beta_0)E_t^t[d_{t+1}] + \beta_1 d_t - p_t R}{\gamma (1 + \beta_0)^2 \sigma^2}$$

$$x_t^{t-1} = \frac{\alpha + (1 + \beta_0)E_t^{t-1}[d_{t+1}] + \beta_1 d_t - p_t R}{\gamma (1 + \beta_0)^2 \sigma^2}.$$

We see that the only difference between the different cohorts trading in the market is their beliefs about future dividends,  $E_t^t[d_{t+1}]$  and  $E_t^{t-1}[d_{t+1}]$ . Since our agents are EBL, their beliefs about future dividend  $d_{t+1}$  are given by

$$E_t^t[d_{t+1}] = d_t,$$

$$E_t^{t-1}[d_{t+1}] = \underbrace{\left(\frac{2^\lambda}{1 + 2^\lambda}\right)}_{w(0,\lambda,1)} d_t + \underbrace{\left(\frac{1}{1 + 2^\lambda}\right)}_{w(1,\lambda,1)} d_{t-1}.$$

These formulas illustrate the mechanics of experience-based learning and the cause of heterogeneity among agents, which will generalize beyond the toy model. Here, the younger generation has only experienced the dividend  $d_t$  and expects the dividends to be identical in the next period. The older generation, having more experience, incorporates the previous dividend in its weighting scheme. Furthermore, we see that belief heterogeneity is increasing in the change in dividends,  $|d_t - d_{t-1}|$ , and decreasing in the recency bias,  $\lambda$ .

Finally, we impose the market clearing condition,  $\frac{1}{2}(x_t^t + x_t^{t-1}) = 1$ , to derive the equilibrium price given these demands. We use the method of undetermined coefficients to solve for  $\{\alpha, \beta_0, \beta_1\}$ . By setting the constants and the terms that multiply  $d_t$  and  $d_{t-1}$  to zero, we obtain the following conditions:

$$R\alpha = \alpha - \gamma (1 + \beta_0)^2 \sigma^2$$

$$R\beta_0 = \frac{1}{2} (1 + \beta_0) \left(1 + \frac{2^\lambda}{1 + 2^\lambda}\right) + \beta_1$$

$$R\beta_1 = \frac{1}{2} (1 + \beta_0) \left(\frac{1}{1 + 2^\lambda}\right).$$

By solving the above system of equations, we obtain the price constant and the loadings of present and past dividends on prices:

$$\alpha = -\frac{\gamma(1 + \beta_0)^2 \sigma^2}{R - 1} \quad (10)$$

$$\beta_0 = \frac{2R^2}{(R - 1) \left(1 + 2R - \frac{2^\lambda}{1+2^\lambda}\right)} - 1 \quad (11)$$

$$\beta_1 = \frac{R \left(1 - \frac{2^\lambda}{1+2^\lambda}\right)}{(R - 1) \left(1 + 2R - \frac{2^\lambda}{1+2^\lambda}\right)}. \quad (12)$$

Solving for these equations is helpful in to order to illustrate how the price loadings on past dividends depend on the magnitude of the recency bias. For instance, as the recency bias increases, prices become more responsive to the most recent dividend,  $\frac{\partial \beta_0}{\partial \lambda} > 0$ , and less responsive to past dividends,  $\frac{\partial \beta_1}{\partial \lambda} < 0$ . The intuition is straightforward: Under higher recency bias, both cohorts put more weight on the most recent dividend realization. Thus, prices become more responsive to recent dividends, and less responsive to past observations. In Section 5, we will present an enriched version of the model with demographic shocks and show how these price loadings vary with the demographic structure of the economy.

The price equations are also useful in that they allow us to derive expressions for price volatility and auto-correlation:

$$\begin{aligned} Var(p_{t+1}) &= \frac{1 + 2R(1 + R + 2^{1+2\lambda}R + 2^{1+\lambda}(1 + R))}{(R - 1)^2(1 + 2(1 + 2^\lambda)R)^2} \sigma^2 \\ Corr(p_t, p_{t+j}) &= \begin{cases} \frac{R(1+R+2^{1+\lambda}R)}{(R-1)^2(1+2(1+2^\lambda)R)^2} & \text{for } j = 1 \\ 0 & \text{for } j > 1, \end{cases} \end{aligned}$$

where  $Var(\cdot)$  and  $Corr(\cdot)$  denote the unconditional variance and correlation respectively. It can be shown, and is intuitive, that the variance of prices is increasing in the recency bias while the price auto-correlation is decreasing in the recency bias. When recency bias is large, beliefs and prices are more dependent on the most recent dividend realization. Since both

agents have observed the most recent dividend, recency bias reduces belief heterogeneity and induces correlated responses to shocks that generates price volatility and reduces price auto-correlation.

One final important aspect of experience-based learning that the toy model allows us to anticipate is return predictability. Since dividends in this model predict prices, they also predict future excess returns:

$$\frac{p_{t+1} + d_{t+1}}{p_t} - R = \frac{\alpha + (1 + \beta_0) d_{t+1} + \beta_1 d_t}{\alpha + \beta_0 d_t + \beta_1 d_{t-1}} - R.$$

Moreover, adding expectation operators  $E_t^t$  and  $E_t^{t-1}$  for generation  $t$  and  $t - 1$ , respectively, dividends also predict expected excess returns. This equation is a first illustration how our model links demographics and market participation (i.e., which generations are trading in the market) to return predictability.

## 4 Results for General Model

We now return to the general case (i.e., allow for any  $q > 1$ ), and characterize the portfolio choices and resulting demands for the risky asset of the different cohorts when agents exhibit experience-based learning. We will impose affine prices, then use market clearing to verify the affine prices guess, and fully characterize demands and prices.

The generalized model will replicate the results from the toy model that prices depend on the dividend payments experienced by the generations trading in the market, on cross-sectional differences in (experience-based) beliefs generating trade volume, and on the volatility and autocorrelation of prices, as well as how these effects are scaled down by the extent of recency bias. In addition, the more general setting allows us to discuss which generations will react more strongly to recent changes in dividends and characterize the abnormal trading volume under various scenarios of dividend changes. We will see how the model generates testable predictions, which we will later bring to the data.

## 4.1 Characterization of Equilibrium Demands

For any  $s, t \in \mathbb{Z}$ , let  $d_{s:t} = (d_s, \dots, d_t)$  denote the history of dividends from time  $s$  up to time  $t$ . For simplicity and WLOG, we assume that the initial wealth of all generations is zero, i.e.,  $W_n^n = 0$  for all  $n \in \mathbb{Z}$ . At time  $t \in \{n, \dots, n_q\}$ , an agent of generation  $n$  determines her demand for the risky asset maximizing  $E_t^n[-\exp(-\gamma x s_{t+1})]$ , as described in (3).

The model set-up allows us to derive a standard expression for risky asset demand:

**Proposition 4.1.** *Suppose  $p_t = \alpha + \sum_{k=0}^K \beta_k d_{t-k}$  with  $\beta_0 \neq -1$ . Then, for any generation  $n \in \mathbb{Z}$  trading in period  $t \in \{n, \dots, n_q\}$ , demands for the risky asset are given by*

$$x_t^n = \frac{E_t^n[s_{t+1}]}{\gamma V[s_{t+1}]} = \frac{E_t^n[s_{t+1}]}{\gamma(1 + \beta_0)^2 \sigma^2}. \quad (13)$$

*Proof.* The result follows by Lemma B.1 in Appendix B. □

## 4.2 Characterization of Equilibrium Prices

The expression for the risky-asset demands in equation (13) allows us to derive equilibrium prices. Note that equation (13) implies that demands at time  $t$  are affine in  $d_{t-K:t}$ . It is easy to see, then, that beliefs about future dividends are linear functions of the dividends observed by each generation participating in the market and that, thus, prices depend on the history of dividends observed by the oldest generation in the market:

**Proposition 4.2.** *The price in any linear equilibrium is affine in the history of dividends observed by the oldest generation participating in the market, i.e., for any  $t \in \mathbb{Z}$*

$$p_t = \alpha + \sum_{k=0}^{q-1} \beta_k d_{t-k}. \quad (14)$$

with

$$\alpha = -\frac{1}{\left(1 - \sum_{j=0}^{q-1} \frac{w_j}{R^{j+1}}\right)^2} \frac{\gamma \sigma^2}{R-1} \quad (15)$$

$$\beta_k = \frac{\sum_{j=0}^{q-1-k} \frac{w_{k+j}}{R^{j+1}}}{1 - \sum_{j=0}^{q-1} \frac{w_j}{R^{j+1}}} \quad k \in \{0, \dots, q-1\} \quad (16)$$

where  $w_k \equiv \frac{1}{q} \sum_{age=0}^{q-1} w(k, \lambda, age)$ .<sup>7</sup>

*Proof.* See Appendix B. □

For each  $k = \{0, 1, \dots, q-1\}$ , one can interpret  $w_k$  as the average weight placed on the dividend observed at time  $t-k$  by all generations trading at time  $t$ . As the formula also reveals, the relative magnitudes of the weights on past dividends,  $\beta_k$ , depend on the extent of agents' recency bias,  $\lambda$ . That is,  $\beta_k$  is a function of  $\lambda$  for all  $k$ . (We typically omit the dependence on  $\lambda$  to ease the notational burden.)

The main idea of the proposition is as follows. In a linear equilibrium, demands at time  $t$  are affine in dividends  $d_{t-K:t}$ . However, from these dividends, only  $d_{t-q+1:t}$  matter for forming beliefs; the dividends  $d_{t-K:t-q}$  only enter through the definition of linear equilibrium. The proof shows that, under market clearing, the coefficients accompanying older dividends  $d_{t-K:t-q}$  are zero. The proposition also implies that we can apply the same restriction to demands and conclude that demands at time  $t$  only depend on  $d_{t-q+1:t}$ .

This result captures the belief channel described by Friedman and Schwartz: Prices are a function of past dividends due to the fact that agents form their beliefs using past realizations of data that they have personally experienced. Our general equilibrium model reveals, however, that the implications of the Friedman and Schwartz view are more nuanced. Since observations of older generations affect current prices, they also affect the demand of younger generations, which did not necessarily experience those observations. In other words, agents use different sets of past data to predict dividends (fundamentals) and prices. To form ex-

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<sup>7</sup> Remember that  $w(k, \lambda, age) \equiv 0$  for all  $k > age$ .



pectations about future dividends agents use the dividends observed during their lifetimes; to form expectations about future prices they look at the history of dividends observed by all cohorts in the market. These insights constitute a novel link between the factors influencing asset prices and demographic composition.

We also observe that  $\frac{\partial \beta_k}{\partial R} < 0$  and  $\frac{\partial \alpha}{\partial R} > 0$  for any  $\lambda$ . Thus, the theory predicts that, if the interest rate is higher, the equilibrium price of the risky asset is higher and less volatile, as the variance of prices is given by  $\sigma_P^2 = \left( \sum_{k=0}^{q-1} \beta_k^2 \right) \sigma^2$ . Furthermore, higher risk aversion  $\gamma$  decreases the equilibrium price by lowering  $\alpha$ .

The following proposition establishes that, as long as agents exhibit any positive recency bias (i.e.,  $\lambda > 0$ ), the sensitivity of prices to past dividends is stronger the more recent the dividend realization.

**Proposition 4.3.** *For  $\lambda > 0$ , more recent dividends affect prices more than less recent dividends, i.e.,  $0 < \beta_{q-1} < \dots < \beta_1 < \beta_0$ .*

*Proof.* See Appendix B. □

This result reflects the fact that the dividends at time  $t$  are observed by all generations whereas past dividends are only observed by older generations. At the same time, the extent to which prices depend on the most recent dividends varies with the level recency bias, as the following Lemma shows:

**Lemma 4.1.** *The effect of the most recent dividend realization on prices,  $\beta_0$ , is increasing in  $\lambda$ , with  $\lim_{\lambda \rightarrow \infty} \beta_0(\lambda) = 1/(Rq - 1)$  and  $\lim_{\lambda \rightarrow \infty} \beta_k(\lambda) = 0$  for  $k > 0$ .*

*Proof.* See Appendix B. □

As  $\lambda \rightarrow \infty$ , the average weights  $w_k$  (defined in Proposition 4.2) converge to  $1_{\{k=0\}}$  for all  $k = 0, 1, \dots, K$ . Therefore,  $\beta_k \rightarrow 0$  for all  $k > 0$  and  $\beta_0 \rightarrow \frac{1}{Rq-1}$ . In other words, under extreme recency bias (i.e.,  $\lambda \rightarrow \infty$ ), only the current dividend affects prices in equilibrium, and does so at its maximum value, while the weights of all past dividends vanish.

These key results on price sensitivity to past dividends, as well as the dampening effect of recency bias on cross-sectional heterogeneity, produce a range of asset pricing implications. We start from two implications of experienced-based learning for the predictability of excess returns and price dynamics that follow immediately.

**Predictability of Excess Returns.** Equilibrium excess returns at time  $t + j$  are given by

$$\frac{p_{t+j+1} + d_{t+j+1}}{p_{t+j}} - R = \frac{\alpha + (1 + \beta_0)d_{t+j+1} + \sum_{k=1}^{q-1} \beta_k d_{t+j+1-k}}{\alpha + \sum_{k=0}^{q-1} \beta_k d_{t+j-k}} - R.$$

Thus, at time  $t$  and for  $j \leq q - 1$ , the past dividends  $d_{t+j-(q-1)}, \dots, d_t$  can be used to predict excess returns. For dividends realized further in the past, i.e., for  $j > q - 1$ , our model predicts that excess returns at time  $t$  are independent from those dividends.

We note that the predictability of excess returns is an equilibrium phenomenon that stems solely from our learning mechanism and not from, say, a built-in dependence on dividends. In fact, a distinguishing feature of our model is that it establishes a link between the age profile of agents participating in the stock market and the factors predicting stock returns. The model connects past realizations to future returns through a nuanced mechanism, namely, the impact of past realizations on the level of disagreements across market participants.

**Price Dynamics.** Our results imply that the variance of prices  $\sigma_P^2$  is given by  $\sigma_P^2 = \sigma^2(\sum_{k=0}^{q-1} \beta_k^2)$ , and that the autocorrelation structure for prices,  $cov(p_{t+j}, p_t)$ , is

$$cov(p_{t+j}, p_t) = \begin{cases} \sigma^2(\sum_{k=0}^{q-1-j} \beta_k \beta_{k+j}) & \text{for any } j \leq q - 1, \\ 0 & \text{otherwise.} \end{cases}$$

A direct implication is that the autocorrelation of prices vanishes as  $\lambda \rightarrow \infty$ . That is, as the recency bias becomes stronger, prices tend to be uncorrelated.

These two sets of results speak to the stylized facts mentioned in the introduction, in particular the Campbell-Shiller finding on the predictability of excess returns using dividends ([Campbell and Shiller \(1988\)](#)). Experience-based learning provides a micro-foundation for

such predictability and, at the same time, limits it to those dividend realizations experienced by the oldest cohorts of traders participating in the market.

Our explanation also generates the prediction that, if two economies differ only in the age range of their stock-market participants, the extent of predictability with past dividends should correspond to those different compositions. We explore the determinants of different cohorts participating to a larger or smaller extent in the stock market

In addition, experience-based learning can provide a micro-foundation for excess volatility, as established by [LeRoy and Porter \(1981\)](#) and [Shiller \(1981\)](#). Similar to prior theoretical approaches, such as the over-extrapolation model of [Barberis et al. \(2015\)](#) and [Barberis et al. \(2016\)](#), our explanation relies on agents' overweighing of recent realization.

### 4.3 Cross-Section of Asset Holdings

The theoretical framework of experience-based learning also allows us to describe long-term consequences of crises via the channel of stock-market participation. In this section, we show that positive shocks (booms) induce a large representation of younger investors in the market while downward shocks (crashes) have the opposite effect.

To illustrate this, we first establish that younger generations react more optimistically than older generations to positive changes in recent dividends, and more pessimistically to negative changes in recent dividends.

**Proposition 4.4.** *For any  $t \in \mathbb{Z}$  and any generations  $n \leq m$  trading at  $t$ , there is a threshold time lag  $k_0 \leq t - m - 1$  such that for dividends that date back up to  $k_0$  periods, the risky-asset demand of the younger generation born at  $m$  responds more strongly to changes than the demand of the older generation born at  $n$ , while for dividends that date back more than  $k_0$  periods the opposite holds. That is,*

1.  $\frac{\partial x_t^m}{\partial d_{t-k}} \geq \frac{\partial x_t^n}{\partial d_{t-k}}$  for  $0 \leq k \leq k_0$  and
2.  $\frac{\partial x_t^m}{\partial d_{t-k}} \leq \frac{\partial x_t^n}{\partial d_{t-k}}$  for  $k_0 < k \leq q - 1$ .

*Proof.* See Appendix B. □

In our model, a younger generation puts more weight on current dividends when forming beliefs an older generation. Hence, when  $d_t$  increases, the younger generations are “overly optimistic” relatively to the older generation; and when current dividends decrease, younger agents are more pessimistic about the return of the risky asset than older ones. This difference is zero only when both agents have the same belief formation. In the proof of Proposition 4.4, we use Lemma 2.1 to extend this intuition to the more recent dividends, as opposed to just the current one. We establish that, for any two cohorts of investors, there is a threshold time lag, up to which past dividends are weighted more by the younger generation, and beyond which past dividend realization are weighted more by the older generation.

We now extent this insight about belief formation and trade reaction into predictions about relative stock-market positions. We show that, as a result of the stronger impact of more recent shocks on the beliefs of younger generations, and the corresponding stronger response in their risky-asset demand, the relative positions of the young and the old in the market fluctuate. Thus, shocks can affect the presence of older versus younger generations in the stock market.

We denote the discrepancy between positions of generations  $n$  and  $n + k$ , in terms of their investment in the risky asset, as  $\xi(n, k, t) \equiv x_t^n - x_t^{n+k}$ . By Proposition 4.1, and some simple algebra, it follows that:

$$\xi(n, k, t) = \frac{E_t^n[\theta] - E_t^{n+k}[\theta]}{\gamma(1 + \beta_0)\sigma^2} \quad (17)$$

for any  $k = \{0, \dots, t - n\}$ . This formulation illustrates that the discrepancy between the positions of different generations is entirely explained by the discrepancy in beliefs. For instance, if for some  $a > 0$ ,  $d_{n:t} \approx d_{n+a:t+a}$ , then  $\xi(n + a, k, t + a) \approx \xi(n, k, t)$ .<sup>8</sup>

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<sup>8</sup> This last claim follows since the inter-temporal change in discrepancies between sets of generations of the same age,  $\xi(n + a, k, t + a) - \xi(n, k, t)$  for  $a > 0$ , is given by  $(\sum_{j=0}^{t-n-k} \{w(j, \lambda, t - n) - w(j, \lambda, t - n - k)\})(d_{t+a-j} - d_{t-j}) + \sum_{j=t-n-k+1}^{t-n} w(j, \lambda, t - n)(d_{t+a-j} - d_{t-j})/(\gamma(1 + \beta_0)\sigma^2)$ .

The next result shows that, among generations born and growing up in “boom times,” understood as periods of increasing dividends, the younger generations have a relatively higher demand for the risky asset than the older generations. The reverse holds for “depression babies,” i.e., generations born during times of contraction. In those times, the younger generations will exhibit a particularly low willingness to invest in the risky asset, relative to older generations born during those times.

**Proposition 4.5.** *Suppose  $\lambda > 0$ . Consider two points in time  $t_0 \leq t_1$  such that dividends are non-decreasing from  $t_0$  up to  $t_1$ . Then for any two generations  $n \leq n+k$  born between  $t_0$  and  $t_1$ , the demand of the older generation for the risky asset ( $x_t^n$ ) is higher than the demand of the younger generation ( $x_t^{n+k}$ ) at any point  $n \leq t \leq t_1$ , i.e.,  $\xi(n, k, t) \leq 0$ . On the other hand, if dividends are non-increasing, then  $\xi(n, k, t) \geq 0$ .*

*Proof.* See Appendix B. □

The proposition illustrates that, while boom times tend to make all cohorts growing up in such times more optimistic, the effect is particularly strong for the younger generations. This induce them to be overrepresented in the market for the risky asset. The opposite holds during times of downturn. All cohorts of investors growing up during such times will be relatively pessimistic about future returns. However, the effect is particularly strong on the younger generation, and as a result they will be underrepresented in the stock market.

#### 4.4 Trade Volume

We now study how learning and disagreements affect the volume of trade observed in the market. We consider the following definition of the total volume of trade in the economy:

$$TV_t \equiv \left( \sum_{n=t-q}^t \frac{1}{q} (x_t^n - x_{t-1}^n)^2 \right)^{\frac{1}{2}} \quad (18)$$

with  $x_{t-1}^t = 0$ . That is, trade volume is the weighted sum (squared) of the change in positions

of all agents in the economy. Using this definition, we can illustrate the link between trade volume and experience-based belief formation.

**Proposition 4.6.** *The trade volume defined in (18) can be expressed as*

$$TV_t = \chi \left\{ \frac{1}{q} \sum_{n=t-q}^t \left[ (\theta_t^n - \theta_{t-1}^n) - \frac{1}{q} \sum_{n=t-q}^t (\theta_t^n - \theta_{t-1}^n) \right]^2 \right\}^{\frac{1}{2}}, \quad (19)$$

where  $\chi = \frac{1}{\gamma\sigma^2(1+\beta_0)}$ .

*Proof.* See Appendix B. □

Expression (19) illustrates that the presence of learning and disagreements induces trade volume through changes in beliefs, which, in our framework, are driven by changes in the observed history of dividends. More specifically, when the change in each cohort's beliefs is different from the average change in beliefs, trade volume increases. Thus, as the formulation shows, trade volume is a function of the volatility of changes in beliefs.

It follows that, to understand the drivers of trade volume, we need to understand the changes in beliefs across cohorts in response to a given shock. In our framework, a shock to dividends impacts the belief of both generations in the market, but the effect on beliefs is stronger for the younger generations. Therefore, an increase in dividends should induce trade if it makes young agents more optimistic than old agents; and a decrease should induce trade if it makes young agents more pessimistic than old agents. This mechanism is solely due to the presence of experience-based learners, since it is essential that each generation reacts differently to the same realization of dividends. If all agents adjust their beliefs equally, trade volume is zero.

We formalize this insight in the following thought experiment capturing the reaction of our economy reaction to a shock that occurs after a long period of stability.

**Thought Experiment.** Suppose  $d_{t_0} = d_{t_0+1} = \dots = d_{t-1} = \bar{d}$  for  $t - t_0 > q$  and that  $d_t \neq \bar{d}$ . Hence, all generations alive at time  $t - 2$  and  $t - 1$  have only observed a constant

stream of dividends,  $\bar{d}$ , over their lifetimes so far. Therefore,  $E_{t-2}^n[d_{t-1}] = E_{t-1}^n[d_t] = \bar{d}$  for all  $n \in \{t-1-q, \dots, t-1\}$  and trade volume in  $t-1$  is zero,  $TV_{t-1} = 0$ .

What happens if, at time  $t$ , a dividend  $d_t \neq \bar{d}$  is observed? In that case, for each generation  $n$  trading at time  $t$ , i.e., for  $n = \{t-q+1, \dots, t\}$ , beliefs are given by  $E_t^n[d_{t+1}] = w(0, \lambda, t-n)(d_t - \bar{d}) + \bar{d}$  which implies the following change in cohort  $n$ 's beliefs:

$$E_t^n[d_{t+1}] - E_{t-1}^n[d_t] = w(0, \lambda, t-n)(d_t - \bar{d}). \quad (20)$$

Trade volume in  $t$  is then given by:

$$TV_t = |d_t - \bar{d}| \chi \left[ \frac{1}{q} \sum_{n=t-q+1}^t \left( w(0, \lambda, t-n)^2 - \left( \frac{1}{q} \sum_{n=t-q+1}^t w(0, \lambda, t-n) \right)^2 \right) \right]^{\frac{1}{2}}. \quad (21)$$

The thought experiments pins down two aspects of the link between the volatility in beliefs and trade volume: First, the trade volume increases proportionally to the change in dividends, independently of whether the change is positive or negative, and proportionally to a function that reflects the dispersion of the weights agents assign to the most recent observation in their belief formation process. Second, the level of trade volume generated by a given change in dividends will depend on the level of recency bias of the economy. For example, as  $\lambda \rightarrow \infty$ , the dispersion in weights decreases as  $w(0, \lambda, age) \rightarrow 1$  for all  $age \in \{0, \dots, q-1\}$ . Thus, our results suggest that higher recency biases (reflected in higher  $\lambda$ ), should generate lower trade volume responses for a given shock to dividends, and vice-versa. This intuition can be seen more precisely in our toy model.

**Trade Volume in Toy Model.** For  $q = 2$ , the trade volume as stated in Proposition 4.6 can be re-written as:

$$TV_t = \frac{\left( \frac{1}{1+2^\lambda} \right) |d_t - d_{t-1}|}{2\gamma\sigma^2(1 + \beta_0)}.$$

In this case, we see that trade volume is decreasing in the recency bias,  $\lambda$ , and it is increasing in the magnitude of the change in dividends,  $|d_t - d_{t-1}|$ .

Overall, experience-based learning not only provides a microfoundation for the existence of positive trade volume, but also generates testable predictions on the cross-sectional differences in trade volume generated by recent dividend shocks.

## 5 Demographics and Equilibrium Prices

The results derived so far illustrate a key feature of experience-based learning: The demographic structure of an economy, and in particular the cross-sectional composition of investors, affect equilibrium prices, demand, and trade volume in a predictable direction. Since experiences vary across cohorts, and since older cohorts have a longer history of experiences than younger cohorts, a given macroeconomic crises will have different long-term consequences when the population structure is different, and we can predict the direction of such differences and calibrate their magnitude.

In this section, we illustrate the link between demographics and asset markets in a world with experience-based learning by considering an unexpected demographics shock rather than a shock to macro outcomes.<sup>9</sup> For example, a war may diminish certain cohorts of a population, while a baby boom increases them.

We use again the simple setting of our toy economy (i.e., with  $q = 2$ ), and denote the mass of young agents at any time  $t$  by  $y_t$ , and the total mass of agents at time  $t$  by  $m_t = y_t + y_{t-1}$ . We assume that  $y_t = y$  and thus  $m_t = 2y = m$  for all  $t < \tau$  and  $t > \tau + 1$ . We then consider the effect of one-time unexpected demographic shock to the mass of agents entering the economy at time  $t = \tau$ . We analyze two types of such shocks: a positive shock,  $y_\tau > y$ , and a negative shock, with  $y_\tau < y$ .

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<sup>9</sup> We have also analyzed the implications of having a growing population, as opposed to a one-time demographic shock. In Online Appendix [OA.3](#), we show that population growth generates a positive trend in prices since the growing mass of agents have increasing demand, and hence prices adjust to equalize supply. Experience-based learning affects the path of the increasing prices. We show that the relative reliance of prices on the most recent dividend is increasing in the population growth rate.



We know from our previous results that, when the demographic structure has equal-sized cohorts and no demographic shock is expected, prices are given by  $p_t = \alpha + \beta_0 d_t + \beta_1 d_{t-1}$ , where  $\{\alpha, \beta_0, \beta_1\}$  are given by equations (10)-(12). This is the case for  $t > \tau + 1$  and, since the shock at time  $\tau$  is assumed to be unexpected, for  $t < \tau$ .<sup>10</sup> For these time periods the economy is as the one described in Section 3. Thus, we are left to characterize demands and prices for  $\tau$  and  $\tau + 1$ , when the generation of the demographic shock is young and old respectively. We make the following guesses:

$$\begin{aligned} p_\tau &= a_\tau + b_{0,\tau} d_\tau + b_{1,\tau} d_{\tau-1} \\ p_{\tau+1} &= a_{\tau+1} + b_{0,\tau+1} d_{\tau+1} + b_{1,\tau+1} d_\tau \end{aligned}$$

We solve the problem by backwards induction. Note that the form of agent's demands remains unchanged. Market clearing in  $\tau + 1$ , with mass  $y$  of young agents and  $y_\tau$  of old agents, implies

$$1 = y \frac{E_{\tau+1}^{\tau+1} [p_{\tau+2} + d_{\tau+2}] - R p_{\tau+1}}{\gamma (1 + \beta_0)^2 \sigma^2} + y_\tau \frac{E_{\tau+1}^\tau [p_{\tau+2} + d_{\tau+2}] - R p_{\tau+1}}{\gamma (1 + \beta_0)^2 \sigma^2}.$$

Our guess is verified, and we obtain the following coefficients for the price function in  $\tau + 1$ :

$$\begin{aligned} a_{\tau+1} &= \alpha \frac{1}{R} \left[ 1 + \frac{R-1}{m_\tau} \right], \\ b_{0,\tau+1} &= \beta_0 \left[ 1 + \frac{1}{R} \left( \frac{m_\tau - y_\tau}{m_\tau} + \frac{y_\tau}{m_\tau} \omega - \frac{y}{m} (1 + \omega) \right) \right] + \frac{1}{R} \left( \frac{m_\tau - y_\tau}{m_\tau} + \frac{y_\tau}{m_\tau} \omega - \frac{y}{n} (1 + \omega) \right), \\ b_{1,\tau+1} &= \beta_1 \frac{y_\tau}{m_\tau} \frac{m}{y}, \end{aligned}$$

where  $\omega \equiv \frac{2^\lambda}{1+2^\lambda}$  and  $m_\tau = y + y_\tau$ . Note that for  $y_\tau = y$ , the coefficients are as those in the baseline model in equations (10)-(12). The above expressions show that the total mass of agents  $m_\tau$  only affects the constant  $a_{\tau+1}$ , while the loadings  $b_{0,\tau+1}$  and  $b_{1,\tau+1}$  are instead

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<sup>10</sup> We use the assumption that the shock is unexpected to construct the series in Figure 3; but none of the result for  $t \geq \tau$  depend on this assumption.

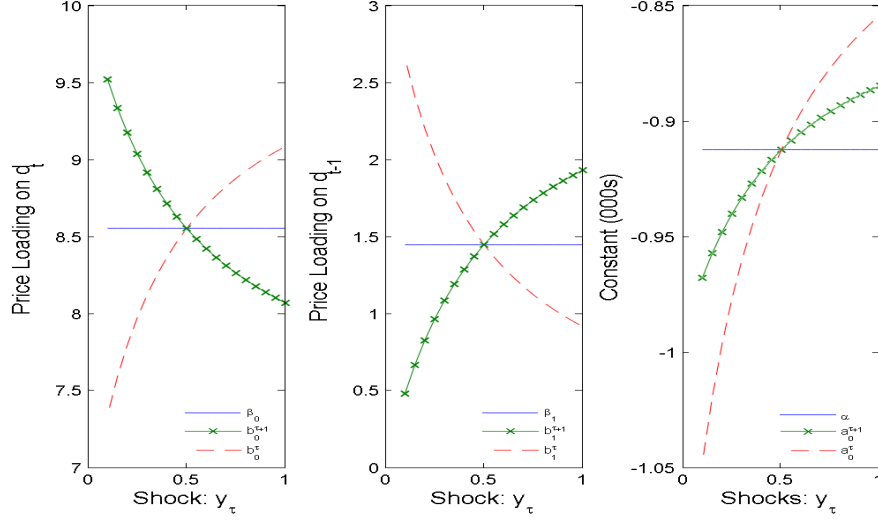


Figure 2: Demographic Shocks and Price Coefficients.

Note: This figure plots coefficients  $\{\beta_0, b_{0,\tau}, b_{0,\tau+1}\}$ ,  $\{\beta_1, b_{1,\tau}, b_{1,\tau+1}\}$ , and  $\{\alpha, a_\tau, a_{\tau+1}\}$ , respectively, as a function of the demographic shock  $y_\tau$ . The results are for  $y = 0.5$ ,  $\gamma = 1$ ,  $\lambda = 3$ ,  $\sigma = 1$ , and  $R = 1.1$ .

functions of the fraction of young agents in the economy. Given this, the demands and market clearing at time  $\tau$  imply:

$$1 = y_\tau \frac{E_\tau^\tau [p_{\tau+1} + d_{\tau+1}] - R p_\tau}{\gamma (1 + b_{0,\tau+1})^2 \sigma^2} + y \frac{E_\tau^{\tau-1} [p_{\tau+1} + d_{\tau+1}] - R p_\tau}{\gamma (1 + b_{0,\tau+1})^2 \sigma^2}$$

Our guess is verified, and we obtain the following coefficients for the price function at  $\tau$ :

$$\begin{aligned} a_\tau &= \frac{1}{R} \left[ a_{\tau+1} - \frac{\gamma (1 + b_{0,\tau+1})^2 \sigma^2}{m_\tau} \right], \\ b_{0,\tau} &= \frac{1}{R} (1 + b_{0,\tau+1}) \left( \frac{y_\tau}{m_\tau} + \frac{m_\tau - y_\tau}{m_\tau} \omega \right) + \frac{1}{R^2} (1 + \beta_0) \frac{y_\tau}{m_\tau} (1 - \omega), \\ b_{1,\tau} &= \frac{1}{R} (1 + b_{0,\tau+1}) \frac{m_\tau - y_\tau}{m_\tau} (1 - \omega). \end{aligned}$$

Figure 2 shows how the the reliance of prices on dividend realizations changes as a function of the size and direction of the demographic shock. From the first two panels, we can see that a positive demographic shock generates a stronger response of prices to the contemporaneous

dividend and a weaker response to past dividends. In addition, note that these responses increase in the size of the demographic shock. The more young people are in the market, who pay no attention to past dividends, the more do current dividends matter and the less do past dividends matter. Consistent with this, we also see that when the  $\tau$ -generation of ‘baby boomers’ becomes old, prices depend less on contemporaneous dividends and more on past dividends than in the baseline case. In addition, the third panel shows that a positive demographic shock generates a level increase in prices that is captured in an increase in the price constant. This effect stems from the higher overall demand for the risky asset since there are more people in the market.

All predictions are reversed for a negative demographic shock, as can be seen at the left side of each graph.

Finally, we consider macroeconomic shocks and demographic shocks simultaneously. That is, we ask how are the model predictions about the market reaction to positive or negative shocks altered when we allow for demographic changes? The implications of experience-based learning under these different scenarios are illustrated in Figure 3. The plot shows how prices and excess returns respond to a positive dividend shock that is contemporaneous to a *Baby-Boom* (increase in cohort size) or to a *War* (decrease in cohort size). The plots assume a positive dividend shock at time  $t = 4$ , which coincides with either a one-time increase (green x-ed line) or a one-time decrease (red dashed line) in cohort size. The solid (blue) line shows the baseline model prediction under no demographic changes.

We can see that for the baby-boom case, prices over-react relative to the baseline case at the time of the shock. This over-reaction is still present, but less severe, when the baby-boom generation is old. The initial over-reaction reflects that, when the baby-boomers are young, there are more people in the market that only pay attention to the present dividends, relative to the baseline case. The milder over-reaction in the period after reflects that, when the generation of baby boomers ages, they update their beliefs and reduce the weight they put on past dividends, and moreover the new generation of young agents, who disregards

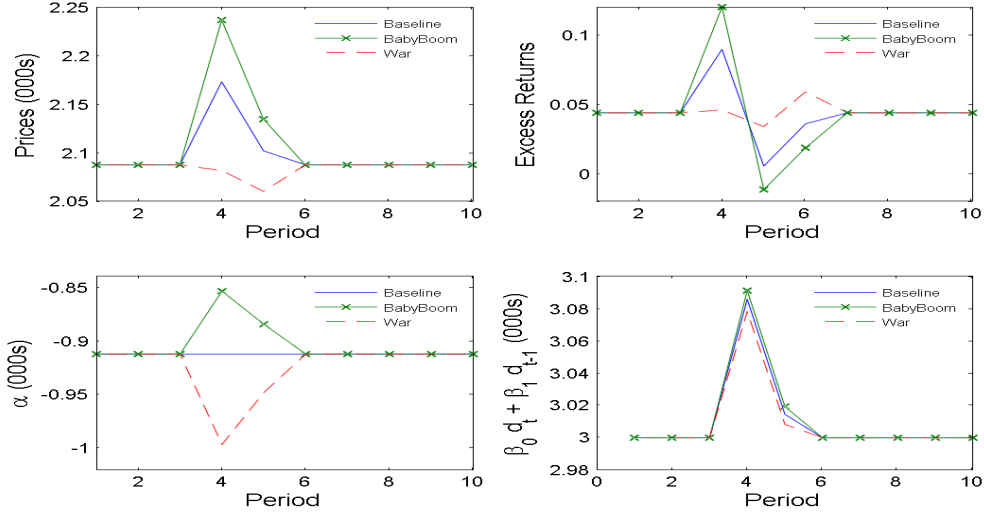


Figure 3: Demographics and Dividend Shock.

Note: This figure plots the response of prices and excess returns to a 1% one-period-only increase in dividends in  $t = 4$ . The results are for  $\gamma = 1$ ,  $\lambda = 3$ ,  $\sigma = 1$ , and  $R = 1.1$ . In the *Baby Boom* line there is a positive demographic shock in  $t = 4$  as well,  $y_4 = 0.75$ , while the *War* line shows a negative demographic shock,  $y_4 = 0.25$ . In the *Baseline* case, there are no demographic shocks,  $y = 0.5$  for all  $t$ .

past dividends, has entered the market. Consistent with these price reactions, returns are positive in response to the shock, and negative following the shock. We can also see that the reaction of prices and returns is reversed when the positive shock occurs during a war. In this particular example, the fall in the overall demand for the risky asset (since less people are present in the market) is strong enough to generate a fall in prices in response to the chosen positive dividend shock.

The results in this section generalize beyond the demographic shocks such as baby booms and wars, which affect cohort size, to shocks affecting the market participation of cohorts, even when their total size is stable. For instance, if other factors increase the market participation of young generations relative to old generations, *ceteris paribus*, the experience-based learning model predicts that the reliance of equilibrium prices on the most recent dividends relative to past dividends goes up. This prediction is in line with the findings

in Cassella and Gulen (2015),<sup>11</sup> who estimate the extent to which investors' recent return experiences (relative to older return experiences) help predict their expectations about future returns. They find a positive relation between this market-wide measure of experience (which they dub 'extrapolation bias') and the relative participation of young versus old investors in the stock market. Furthermore, they find that when the level of extrapolation bias is high in the market, the predictive power of the price-dividend ratio for future returns goes up. While the ideal test of our model would exploit an endogeneous shock to market participation, their results are nevertheless consistent with and suggestive of experience-based learning.

In the next section, we will present several additional empirical findings that are in line with the predictions of our theoretical model.

## 6 Stylized Empirical Facts

As we have argued in the paper so far, our model of experience-based learning generates a host of testable predictions about prices, price sensitivities, trade volume, and stock market participation.

We now examine whether our theoretical model is consistent with aggregate empirical facts about equity holdings and stock turnover.

While some predictions are harder to test, or generate predictions that can be attributed to several explanations (e.g., heigher weights on the most recent dividend, relative to previous dividends, in determining prices),

The model generates at least two sets of predictions that are directly testable: First, cross-sectional differences in the demand for the risky asset reflect cross-sectional differences in lifetime experiences of risky-asset payoffs. And second, the larger the cross-sectional differences in experience-based beliefs are, the higher is the trade volume.

To test these model implications, we combine historical data on stock-market performance, obtained from Robert Shiller's website, with data on stock holdings from the Survey of

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<sup>11</sup> We thank Nick Barberis and Huseyin Gulen for the suggestion.

Consumer Finances (SCF) and stock turnover data from the Center for Research in Security Prices (CRSP).

The key variable to construct for both sets of predictions is a measure of lifetime experiences of dividends. Dividends in our model, however, do not translate one-to-one to the dividend payments recorded in CRSP. Theoretically, dividends in the Lucas-tree economy capture news about firm performance. Empirically, firms have incentives to smooth dividends, or they may retain earnings rather than distribute them to shareholders. We therefore turn to stock market returns, rather than dividend payments, as a measure of risky asset returns. We calculate the lifetime experiences of the different generations as the weighted average of the annual SP500 return over their lifetimes so far. Alternatively, we employ the SP500 Index and add the weighted sum of dividend payments. Given the increasing time trend of dividends, we de-trend all performance measures using the consumer price index (CPI). Applying the formula from equation (5), we employ both linearly declining weights and a steeper weighting function (with  $\lambda = 3$ ), corresponding to the range of empirical estimates in [Malmendier and Nagel \(2011\)](#).

We construct two measures of differences in lifetime experiences across cohorts. Our first measure is the difference between the lifetime experiences of older generations (60 years and older) minus those of younger generations (40 years and younger). For both age groups, the experienced performance is calculated as the weighted average of each cohort within that age range, where each cohort-year is weighted by the total number of people of that cohort in that year.<sup>12</sup> Our second measure approximates disagreement across cohorts in a given year by the standard deviation of their lifetime experiences. We again use the total number of individuals in each cohort in a given year to weight the observations.

To test the first prediction, we relate the differences in lifetime experiences between older versus younger cohorts to the differences in their stock market investment. Our source of

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<sup>12</sup> We include birth-year returns in the experience measure. That is, we construct the measure as if all individuals in the survey were born on January 1 and the SCF were always conducted on January 1. The results are robust to different assumptions, e.g., the assumption that either birthday or date of survey or both are assumed to be December 31.

household-level microdata is the Survey of Consumer Finances (SCF), which provides repeated cross-section observations on asset holdings and various household background characteristics. Our variable construction follows [Malmendier and Nagel \(2011\)](#). We use all waves of the modern, triannual SCF, available from the Board of Governors of the Federal Reserve System since 1983. In addition, we employ some waves of the precursor survey, available from the Inter-university Consortium for Political and Social Research at the University of Michigan. The precursor survey starts in 1947, but includes age (rather than 5- or 10-year brackets) only since 1960. We use all survey waves that also offer stock-market participation information, i.e., the 1960, 1962, 1963, 1964, 1967, 1968, 1969, 1970, 1971, and 1977 surveys.

For the extensive margin of stock holdings, we construct a binary variable for stock-market participation. It indicates whether a household holds more than zero dollars worth of stocks. We define stock holdings as the sum of directly held stocks (including stock held through investment clubs) and the equity portion of mutual fund holdings, including stocks held in retirement accounts (e.g., IRA, Keogh, and 401(k) plans).<sup>13</sup>

For the intensive margin of stock holdings, we calculate the fraction of liquid assets invested in stocks as the share of directly held stocks plus the equity share of mutual funds can be calculated in all surveys from 1960-2013 other than 1971. Liquid assets are defined as stock holdings plus bonds plus cash and short-term instruments (checking and savings accounts, money market mutual funds, certificates of deposit). In those analyses, we drop all households that have no money in stocks.

We aggregate the households into the aforementioned age groups by taking the unweighted average of the intensive and extensive margin over all households whose head falls into that particular age-group. Note that the sample selection of the survey has changed over time. Both the intensive and extensive margin are weighted by sample weights included in the SCF.

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<sup>13</sup> For 1983 and 1986, we need to impute the stock component of retirement assets from the type of the account or the institution at which they are held and allocation information from 1989. From 1989 to 2004, the SCF offers only coarse information on retirement assets (e.g., mostly stocks, mostly interest bearing, or split), and we follow a refined version of the Federal Reserve Board’s conventions in assigning portfolio shares. See [Malmendier and Nagel \(2011\)](#) for more details.

The weighted estimates are representative of the U.S. population.<sup>14</sup>

Figure 4 depicts the relationship between the extensive and intensive margin of stock holdings and experienced stock-market returns, by plotting the differences between the above-60 age-group and the below-40 age-group for both variables. Experienced returns are calculated from equation (5), with  $\lambda$  set to 1 (corresponding to linearly declining weights) in graphs 4(a) and 4(c), and with  $\lambda = 3$  (corresponding more closely to previous estimates of lambda from Malmendier and Nagel (2011)) in graphs 4(b) and 4(d).

For all specifications, we find that the empirical results are in line with the model predictions. Graph 4(a) shows that the older age-group is more likely to hold stock compared to the younger age-group when they have experienced higher stock-market returns in their lives, and the opposite hold when the lifetime average of the younger generations is higher than that of the older generations. The steepness of the weighting function, and hence the extent of imposed recency bias appears to make little difference, as the comparison with graph 4(b) for  $\lambda = 3$  reveals.

The same holds for the intensive margin of stock-market investment. Both graph 4(c) and graph 4(d) indicate that older generations invest a relatively higher share of their liquid assets into stocks vis-à-vis the younger generations when their experienced returns have been higher than those of the younger age-group over their respective life-spans so far, and that the opposite holds when they have experienced lower returns than the younger cohorts.

All results are similar when we use the other performance measure, which adds actual dividend payments to experienced returns.

Turning to the second prediction, our model also suggests that trade volume is high when disagreement among investors is high. To test this second prediction, we examine the co-movement between (abnormal) trade volume and the standard deviation of experienced stock returns, i.e., the aforementioned measure of disagreement.

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<sup>14</sup> The 1983-2013 waves of the SCF oversample high-income households with significant stock holdings. The oversampling is helpful for our analysis of asset allocation, but could also induce selection bias. By weighting the data using SCF sample weights, we undo the overweighting of high-income households and also adjust for non-response bias.





(a) Stock-market participation ( $\lambda = 1$ )



(b) Stock-market participation ( $\lambda = 3$ )



(c) Fraction invested in stock ( $\lambda = 1$ )



(d) Fraction invested in stock ( $\lambda = 3$ )

Figure 4: Experienced Returns and Stock Holding

*Difference in experienced returns* is calculated as the lifetime average experienced returns of the SP500 Index, using declining weights either linearly ( $\lambda = 1$ ) or super-linearly ( $\lambda = 3$ ) as in equation (5). *Stock-market participation* is measured as the fraction of households in the respective age groups that hold at least \$1 of stock ownership, either as directly held stock or indirectly, e.g. via mutuals or retirement accounts. *Fraction invested in stock* is the fraction of liquid assets stock-market participants invest in the stock market. We classify households whose head is aged 60 or older as “old,” and households whose head is younger than 40 as “young.” Difference in stockholdings, the y-axis in graphs (a) and (b), is calculated as the difference between the logs of the fractions of stock holders among the old and among the young age group. Percentage stock, the y-axis in graphs (c) and (d), is the difference in the fraction of liquid assets invested in stock. The red line depicts the linear trend.

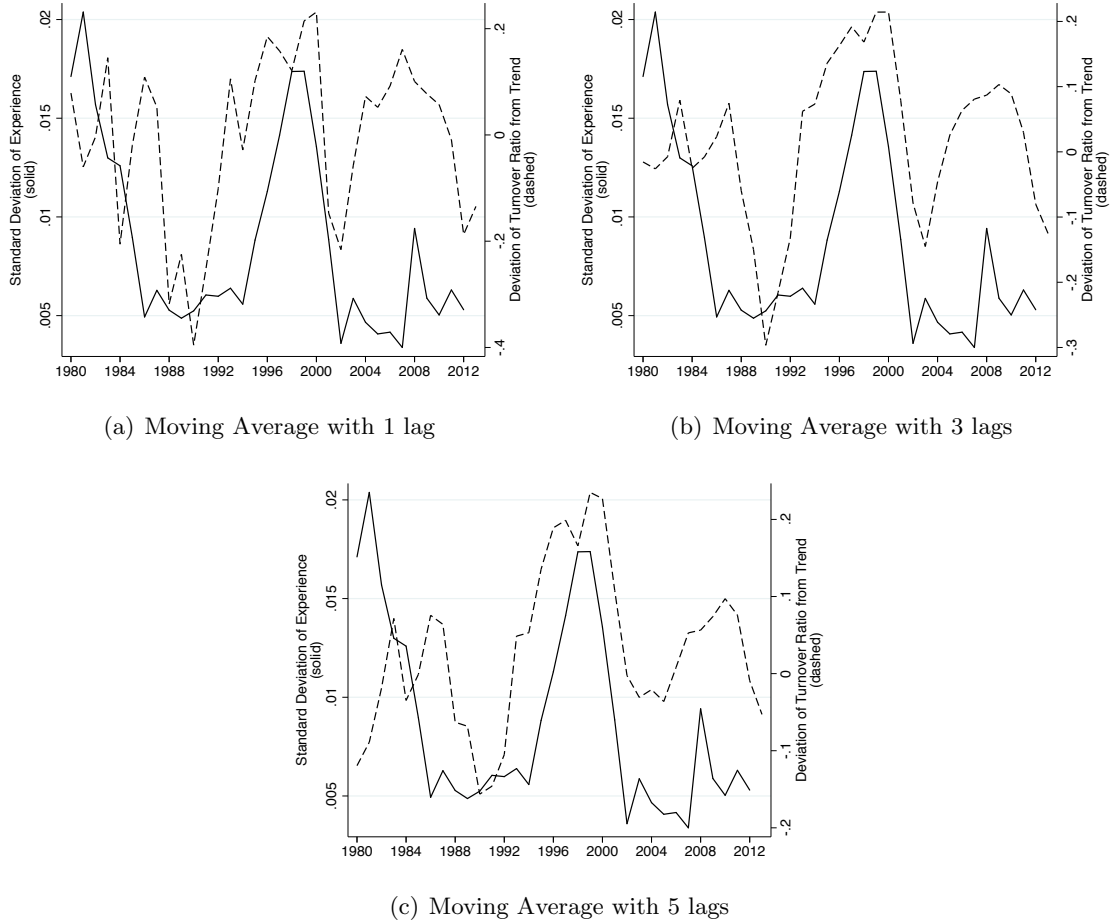


Figure 5: Turnover Ratio and Disagreement in Experienced Returns

The solid line shows the standard deviation of the lifetime average stock-return experiences of all cohorts in a given year, where we weight each age-group with the number of persons of that age. The dashed line depicts the deviation of the turnover ratio from its trend. The turnover ratio is calculated as the ratio of the value of traded stocks and the value of stocks outstanding. We de-trend the turnover ratio by first taking the log and then removing a linear trend. Graphs (a), (b), and (c) the moving averages of the turnover ratio with one, three, and five lags, respectively.

As a measure of abnormal trade volume we calculate the deviation of the turnover ratio from its trend as follows. First we compute trading volume from monthly data on the number of traded shares, the number of outstanding shares, and stock prices for all ordinary common shares in CRSP from 1960 to 2007. For each month, we multiply the number of traded shares with the average share price. To account for changes in capitalization, we scale the trade volume by market capitalization (i.e., by the number of shares outstanding times average stock price). We refer to the scaled trade volume as the turnover ratio. To align the turnover ratio with the frequency of our disagreement variable, we calculate the annual turnover ratio as the average of the monthly turnover ratios. Finally, we detrend the turnover ratio to adjust for time-series factors that affect trade volume but are not related to disagreement, such as technological progress. To de-trend the turnover ratio, we first take the log and then remove a linear trend. That is, we regress the logged turnover series on a linear time trend, and then average the residuals for each year to obtain a measure of deviation of the trading activity from the trend.

Graphs (a), (b), and (c) of Figure 5 display the moving averages of the detrended turnover ratio with one, three, and five lags, respectively. Each graph also displays the experience-based disagreement across cohorts, approximated by the standard deviation of experienced performance. As the figure shows, the turnover ratio tends to be higher than the turnover trend when experienced-based disagreement among investors is high. These results imply that different views about future stock returns, resulting from different lifetime experiences, affect trading behavior of different cohorts of investors.

## 7 Extension: Non-Myopic Agents

In this section, we consider the model with non-myopic agents who consume in the last period of their lives. As a result, in each period agents make their portfolio decisions to maximize the expected utility of consuming their final wealth. First, we characterize the demands for the risky asset of different cohorts in the economy. In contrast to the myopic case, now agents

need to account for the dynamic nature of their portfolio problem. Second, we characterize equilibrium prices, and show that the result of Proposition 4.2 continues to hold with non-myopic agents: Prices are affine functions of past dividends observed by the generations that are trading. Finally, we focus on the two-generations model,  $q = 2$ , to compare the remaining findings with those of the baseline model with myopic agents.

## 7.1 Characterization of risky demands for non-myopic agents

For any  $s, t \in \mathbb{Z}$ , let  $d_{s:t} = (d_s, \dots, d_t)$  denote the history of dividends from time  $s$  up to time  $t$ . At time  $n$ , a  $n$ -generation agent solves the following problem:

$$\max_{\mathbf{x} \in \mathbb{R}^q} E_n^n [-\exp(-\gamma W_{n+q}^n(\mathbf{x}))] \quad (22)$$

$$\text{s.t. } W_{n+q}^n(\mathbf{x}) = \sum_{\tau=n}^{n_q} R^{n_q-\tau} x_\tau s_{\tau+1} \quad (23)$$

where  $\mathbf{x} \in \mathbb{R}^q$  are the  $q$  trading decisions from  $n$  up to  $n_q$ . Note that, by moving from maximizing next period's wealth under the myopic formulation to maximizing final-period wealth, the non-myopic formulation introduces discounting with factor  $R^{n_q-\tau}$ .

We continue to assume that the initial wealth of all generations is zero, i.e.,  $W_n^n = 0, \forall n$ . We can cast this problem iteratively — by solving from  $n_q = n + q - 1$  backwards — as

$$V_{n_q}^n(d_{n_q-K:n_q}) = \max_{x \in \mathbb{R}} E_{n_q}^n [-\exp(-\gamma s_{n+q}x)] \text{ and} \quad (24)$$

$$V_\tau^n(d_{\tau-K:\tau}) = \max_{x \in \mathbb{R}} E_\tau^n [V_{\tau+1}^n(d_{\tau+1-K:\tau+1}) \exp(-\gamma s_{\tau+1}x)], \forall \tau \in \{n, \dots, n_q - 1\} \quad (25)$$

**Remark 7.1.** Notice that  $V_\tau^n$  does not include the wealth at time  $\tau$ , that is, from equation (24), the optimization problem can be cast as  $\max_{x \in \mathbb{R}} \exp\{-\gamma RW_{n_q}^n\} E_{n_q}^n [-\exp(-\gamma s_{n+q}x)]$ . However, our definition of  $V_{n_q}^n$  omits the term  $\exp\{-\gamma RW_{n_q}^n\}$  since it does not affect the maximization.

This shows that, although the  $n$ -generation's problem at  $n_q$  is a static portfolio problem, for

any other  $\tau \in \{n, \dots, n_q - 1\}$ , it is not because  $V_{\tau+1}^n$  is correlated with  $s_{\tau+1}$  through dividends. That is, dividend realization  $d_{\tau+1}$  impacts (i) the net payoff obtained from investing  $x_\tau$  in the risky asset at time  $\tau$ , and (ii) the continuation value  $V_{\tau+1}^n(d_{\tau+1-K:\tau+1})$  by affecting the beliefs of the  $n$ -generation at  $\tau + 1$ , and the resulting portfolio decision.

We show that the dynamic portfolio problem of agents in this economy cannot be expressed as a succession of static problems, as is standard in the literature (see [Vives \(2010\)](#)). This is due to the presence of learning and to the fact that agents understand how their beliefs evolve over their lifetime. These features introduce a correlation between future returns and continuation values that distorts the portfolio decisions. In what follows, however, we show that the agents dynamic portfolio problem can be expressed as an *adjusted static* problem where dividends follow a normal distribution with *adjusted* mean and variance. Intuitively, agents recognize that very high and very low realizations of future dividends will lead to more disagreement, which they will exploit in their future trades. As a result, extreme realizations are now associated with higher continuation values, leading to a downward adjustment of the variance.

Let  $E_{N(\mu, \sigma^2)}[\cdot]$  and  $V_{N(\mu, \sigma^2)}[\cdot]$  be the expectation and variance with respect to a Gaussian pdf with mean  $\mu$  and  $\sigma^2$ .

**Proposition 7.1.** *Suppose  $p_t = \alpha + \sum_{k=0}^K \beta_k d_{t-k}$  with  $\beta_0 \neq -1$ . Then, for any generation  $n$  in period  $n + j$  for  $j \in \{0, \dots, q - 1\}$  (the age of the generation), demands for the risky asset are given by:*

$$x_{n+j}^n = \frac{E_{N(m_j, \sigma_j^2)}[s_{n+j+1}]}{\gamma R^{q-1-j} V_{N(m_j, \sigma_j^2)}[s_{n+j+1}]} \quad (26)$$

where:

$$m_j \equiv \frac{\theta_{n+j}^n - \sigma^2 \left( b_j + \sum_{k=1}^K b_j(k) d_{n+j-k} \right)}{2c_j \sigma^2 + 1} \quad (27)$$

$$\sigma_j^2 \equiv \frac{\sigma^2}{2c_j \sigma^2 + 1} \quad (28)$$

for  $\{\{b_j(k)\}_{k=1}^{q-1}, b_j, c_j\}$  constants that change with the agent's age ( $j$ ) (for exact expressions see the proof).

*Proof.* See Appendix C. □

The intuition of the proof is as follows. By solving the problem backwards we note that at time  $n_q$  the problem is in fact a static one (see equation (24)). In particular we show that  $V_{n_q}^n$  is of the form exponential-quadratic in  $d_{n_q}$  (see Lemma B.1 in the Appendix). We then show that the exponential-quadratic term times the Gaussian distribution of dividends imply a new Gaussian distribution with an slanted mean and variance (see Lemma C.1 in the Appendix). Thus the problem at time  $n_q - 1$  can be viewed as a static problem with a modified Gaussian distribution, and consequently (a) demands are of the form of 26 and  $V_{n_q-1}^n$  is also of the exponential-quadratic form. The process thus continues until time  $n$ .

After straightforward algebra, we can cast equation (26), as <sup>15</sup>

$$\begin{aligned} x_{n+j}^n &= \frac{1}{R^{q-1-j}} \frac{E_{N(\theta_{n+j}^n, \sigma^2)}[s_{n+j+1}]}{\gamma V_{N(\theta_{n+j}^n, \sigma^2)}[s_{n+j+1}]} - \frac{(b_j + \sum_{k=1}^K b_j(k) d_{n+j-k})}{\gamma R^{q-1-j} (1 + \beta_0)} \\ &\equiv \frac{1}{R^{q-1-j}} \tilde{x}_{n+j}^n + \Delta_{n+j}^n \end{aligned} \quad (29)$$

The term  $\tilde{x}_{n+j}^n$  coincides with the demand of a *static* portfolio problem for an agent with beliefs  $\theta_{n+j}^n$ ; see Proposition 4.1. We coin this term the *myopic component* of the demand for risky assets. The scaling by  $1/R^{q-1-j}$  arise because agents discount the future by  $R$ . The second term  $\Delta_{n+j}^n \equiv -\frac{(b_j + \sum_{k=1}^K b_j(k) d_{n+j-k})}{\gamma R^{q-1-j} (1 + \beta_0)}$ , is an adjustment which accounts for the dynamic nature of the problem, and thus, we call it the *dynamic component*. It arises because agents understand that they are learning about the risky asset, and thus understand that the value function is correlated with the one-period-ahead returns.

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<sup>15</sup> Note that  $E_{N(b+a, s)}[s_{n+1}] = E_{N(a, s)}[s_{n+1}] + (1 + \beta_0)b$ .

## 7.2 Characterization of equilibrium prices for non-myopic agents

The following proposition shows that in a linear equilibrium prices at any time  $t$  only depend on the dividends observed by the generations trading at time  $t$ . This result shows that the insights in Proposition 4.2 continue to hold in this setup with non-myopic agents.

**Proposition 7.2.** *For  $R > 1$ , the price in any linear equilibrium with  $\beta_0 \neq -1$  is affine in the history of dividends observed by the oldest generation participating in the market. For any  $t \in \mathbb{Z}, q \geq 1$ ,*<sup>16</sup>

$$p_t = \alpha + \sum_{k=0}^{q-1} \beta_k d_{t-k}. \quad (30)$$

*Proof.* See Appendix C. □

The proof follows along the same lines as the one for the myopic case.

## 7.3 Illustration: The $q = 2$ Case

We now specialize our results to the case with  $q = 2$ . By doing so, we are able to sharpen our previous results regarding the behavior of prices and risky demands in equilibrium.

In Online Appendix OA.2, Lemma OA.2.1, we establish that the coefficients on the price function,  $\{\alpha, \beta_0, \beta_1\}$ , solve a complicated system of linear equations. With this result, we are able to establish that prices react positively to dividends  $d_t$  and  $d_{t-1}$ . Formally,

**Proposition 7.3.** *For  $\lambda > 0$ ,  $\alpha \leq 0$  and  $0 < \beta_1 < R\beta_0$ .*

*Proof.* See Appendix C. □

This proposition is analogous to Proposition 4.3 and establishes that when agents form their beliefs by using non-decreasing weights (i.e.,  $\lambda \geq 0$ )  $\beta_0 R$  is larger than  $\beta_1$ . This result reflects the fact that the dividends at time  $t$  are observed by both generations whereas  $d_{t-1}$

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<sup>16</sup> Heuristically, an equilibrium with  $\beta_0 = -1$  is not well-defined since in this case the excess payoff, say,  $s_{t+q-1}$  is deterministic given the information at time  $t + q - 2$  and thus the agent will take infinite positions depending on  $d_{t+q-1} + p_{t+q-1} - rp_{t+q-2}$ .

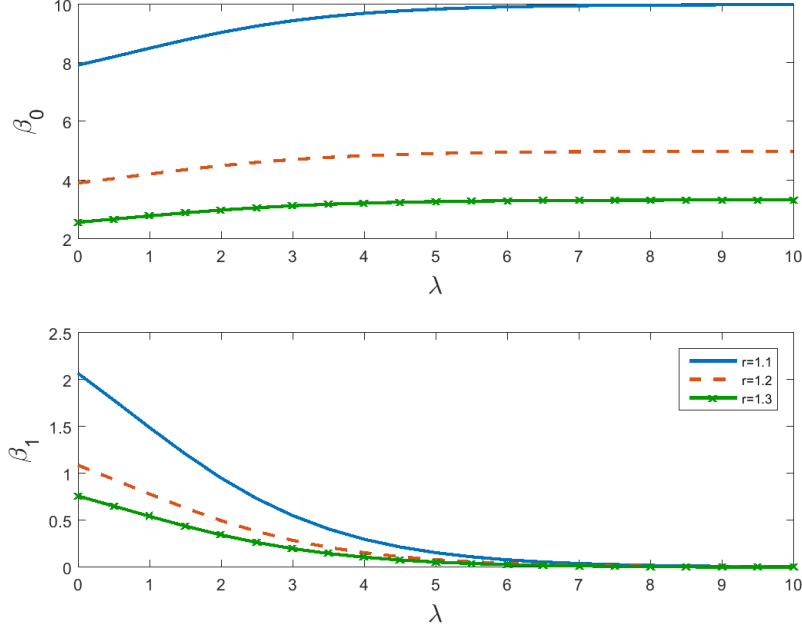


Figure 6: Comparative Statics: Sensitivity of Prices to Dividends for the  $q=2$  Case.

is only observed by the old generation; in fact it is not hard to see from the equations that in the case  $w(1, \lambda, 0) = 0$  –agents do not put any weight on the previous dividend,– then  $\beta_1 = 0$ .

Figure 6 depicts the behavior of  $\{\beta_0, \beta_1\}$  for different values of  $(\lambda, R)$ . Note that the values of  $\{\beta_0, \beta_1\}$  are independent of the process for dividends,  $\sigma^2$ , and of the coefficient of risk aversion,  $\gamma$ . Thus, the results shown in the figure do not depend on parameter values other than the ones used for comparative statics:  $(\lambda, R)$ .

The next proposition establishes that, as it was the case with myopic agents (Proposition 4.4), the demand of the young generation (decreases) increases, while the one of the old generation (increases) decreases, when current dividends (decrease) increase. The opposite result holds for the previous period dividends.

**Proposition 7.4.** *For  $\lambda > 0$ : (1)  $\frac{\partial x_t^t}{\partial d_t} > 0 > \frac{\partial x_t^{t-1}}{\partial d_t}$ , and (2)  $\frac{\partial x_t^t}{\partial d_{t-1}} < 0 < \frac{\partial x_t^{t-1}}{\partial d_{t-1}}$ .*

*Proof.* See Appendix C. □



In our model, the young generation puts more weight on current dividends when forming beliefs, so when  $d_t$  increase, they young are “overly optimistic” relatively to the old generation. This effect contributes to the result (1) (and similar reasoning contributes to results (2)); however, this is not the only effect to consider. There additional effects due to the fact that the young are confronted with a different horizon investment.

In order to shed some light on the different effects, recall that in equation (29) we decomposed the demand for risky asset into two components: The myopic one and the dynamic one. For the particular case of  $q = 2$  these decomposition yields  $x_t^{t-1} = \tilde{x}_t^{t-1}$  and  $x_t^t = \tilde{x}_t^t + \Delta_t^t$ .

First, we analyze how dividends affect the myopic component of demands. Let

$$\frac{\partial (\tilde{x}_t^t - \tilde{x}_t^{t-1})}{\partial d_t} = \underbrace{\frac{(1 + \beta_0)(1 - w(0, \lambda, 1))}{\gamma(1 + \beta_0)^2 \sigma^2}}_{\text{Beliefs Term}} + \underbrace{\frac{1 + \beta_0 + \beta_1 - R\beta_0}{\gamma(1 + \beta_0)^2 \sigma^2} \left( -\frac{R-1}{R} \right)}_{\text{Discount Term}}$$

We refer to the first term as the *Beliefs Term*. This term is positive, and it reflects that an increase (decrease) in dividends makes young agents more optimistic (pessimistic) about the return of the risky asset than adult agents because the put more weight on recent realizations. This term is zero when both agents have the same belief formation (e.g.  $w(0, \lambda, 1) = 1$ ). The second term is the *Discount Term*, which is negative (see Lemma C.5 in Appendix). Even when agents share beliefs, young agents react less aggressively to a change in dividends (in their beliefs) because they discount the future more than old agents since  $R > 1$ .

Regarding the dynamic term, observe that

$$\frac{\partial \Delta_t^t}{\partial d_t} = \frac{\beta_1 - R\beta_0}{\gamma(1 + \beta_0)^2 \sigma^2} \frac{1}{R} \left( \frac{\sigma^2}{s^2} - 1 \right) - \frac{(1 + \beta_0)}{\gamma(1 + \beta_0)^2 \sigma^2} \left( \frac{l(1, 1)l(0, 1)}{(1 + \beta_0)^2 R} \right)$$

where  $s^2 = \sigma^2 \frac{(1 + \beta_0)^2}{(1 + \beta_0)^2 + ((1 + \beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0)^2}$ . Because the first term in the RHS is negative but the second term is positive (see proof of Proposition 7.4), we can not pin down the sign of  $\frac{\partial \Delta_t^t}{\partial d_t}$ . However, in Proposition 7.4, we are able to show that the belief term always dominates; that is,  $\frac{\partial (x_t^t - x_t^{t-1})}{\partial d_t}$  is positive. In figure 7 we show the behavior of each of the terms for

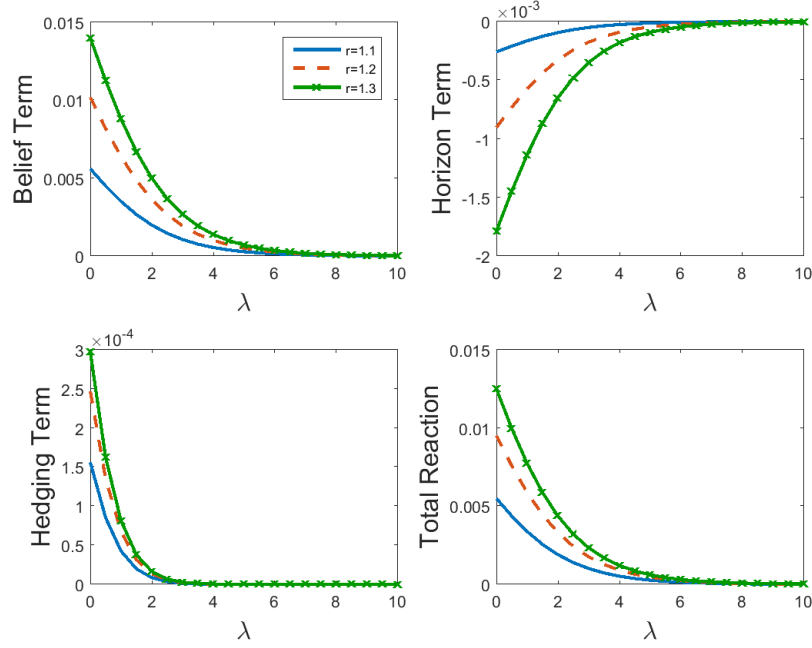


Figure 7: Comparative Statics: Sensitivity of Demands to Dividends for the  $q=2$  Case.

Decomposition of  $\frac{\partial(x_t^t - x_t^{t-1})}{\partial d_t}$  into the Belief, Horizon, and Hedging Terms.

different values of  $(R, \lambda)$ . Importantly, as  $\lambda$  increases, the “old” generation puts less weight to past dividends, and thus the belief heterogeneity vanishes.

## 8 Conclusion

In this paper, we have proposed a simple OLG general equilibrium framework to study the effect of personal experiences of macroeconomic shocks on future economic outcomes such as the cross-section of asset holdings, asset prices, and market volatility. We have done so by incorporating the two main empirical features of experience effects, the over-weighing of lifetime experiences and recency bias, into the belief formation process of agents. We find that through our mechanism, macroeconomic shocks can have long-lasting effects on an economy, as suggested by [Friedman and Schwartz \(1963\)](#) and [Blanchard \(2012\)](#).

We highlight two channels through which shocks have long-lasting effects on economic outcomes. The first is the belief formation process, since all agents update their beliefs about the future after observing a given shock. The second is the cross-sectional heterogeneity in the population, since different experiences generate belief heterogeneity. Furthermore, we show that the demographic composition of an economy has important implications for the extent to which macroeconomic shocks can have long-lasting effects through the above described channels. Most importantly, we take our model predictions to the data, and find that they are consistent with empirical stylized facts on portfolio decisions and trade volume.

The results of this paper underline the importance of formally modeling the belief formation process of agents. This is not only relevant for improving our understanding of economic behavior, but also for effective policy making. We believe that the next step is two-fold. First, we need to continue improving our understanding of how agents form their beliefs about future economic outcomes. Second, it is important that these findings are formalized and incorporated to standard models to continue shaping our understanding of the way economies operate.

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## Appendix A Proofs for Results in Section 2

*Proof of Lemma 2.1.* Let  $\Delta(k) \equiv w(k, \lambda, age) - w(k, \lambda, age')$  for all  $k \in \{0, \dots, age\}$ . We need to show that  $\exists k_0 \in \{0, \dots, age'\}$  such that  $\Delta(k) < 0$  for all  $k \leq k_0$ , and  $\Delta(k) \geq 0$  for all  $k > k_0$ , with the last inequality holding strictly for some  $k$ .

For  $k > age'$ ,  $\Delta(k) > 0$  since  $w(k, \lambda, age') \equiv 0$ , and hence  $\Delta(k) = w(k, \lambda, age) > 0$ , for all  $k \in \{age' + 1, \dots, age\}$ .

For  $k \leq age'$ , we note that

$$\Delta(k) > 0 \iff Q(k) := \frac{w(k, \lambda, age)}{w(k, \lambda, age')} > 1. \quad (31)$$

Hence, it remains to be shown that  $\exists k_0 \in \{0, \dots, age'\}$  such that  $Q(k) < 1$  for all  $k \leq k_0$ , and  $Q(k) \geq 1$  for all  $k > k_0$ . Since the normalizing constants used in the weights  $w(k, \lambda, age)$  are independent of  $k$  (see the definition in (5)), we absorb them in a constant  $c \in \mathbb{R}^+$  and rewrite

$$Q(k) = c \cdot \frac{(age + 1 - k)^\lambda}{(age' + 1 - k)^\lambda} = c \cdot \left[ \frac{age + 1 - k}{age' + 1 - k} \right]^\lambda = c \cdot \alpha(k)^\lambda \quad \forall k \in \{0, \dots, age'\}. \quad (32)$$

The function  $x \mapsto \alpha(x) = \frac{age+1-x}{age'+1-x}$  has derivative  $\alpha'(x) = \frac{age-age'}{(age'+1-x)^2} > 0$  for  $x \in [0, age' + 1)$ , and hence  $Q(\cdot)$  is strictly increasing over  $\{0, \dots, age'\}$ . Thus, to complete the proof, we only have to show that  $Q(k) < 1$  or, equivalently,  $\Delta(k) < 0$  for some  $k \in \{0, \dots, age'\}$ . We know that  $\sum_{k=0}^{age} \Delta(k) = 0$  because  $\sum_{k=0}^{age} w(k, \lambda, age) = \sum_{k=0}^{age'} w(k, \lambda, age') = 1$ , and we also know that  $\sum_{k=age'+1}^{age} \Delta(k) > 0$  since  $\Delta(k) = w(k, \lambda, age) > 0$  for all  $k \in \{age' + 1, \dots, age\}$ . Hence, it must be that  $\Delta(k) < 0$  for some  $k < age'$ .  $\square$

## Appendix B Proofs for Results in Section 4

Proposition 4.1 directly follows from the following Lemma.

**Lemma B.1.** *Let  $z \sim N(\mu, \sigma^2)$ , then for any  $a > 0$ ,*

$$x^* = \arg \max_x E[-\exp\{-axz\}] = \frac{\mu}{a\sigma^2}$$

and

$$\max_x E[-\exp\{-axz\}] = -\exp\left\{-\frac{1}{2}(\sigma ax^*)^2\right\} = -\exp\left(-\frac{1}{2}\frac{\mu^2}{\sigma^2}\right).$$

*Proof.* See Online Appendix OA.1.1.  $\square$

*Proof of Proposition 4.2.* We show the result for the guess  $p_t = \alpha + \beta_0 d_t + \dots + \beta_K d_{t-K}$  with  $K = q$ . This case shows the logic of the proof; the proof for the case starting from an arbitrary lag  $K \geq q$  is analogous but more involved, and omitted for simplicity.

From Lemma B.1, agents' demand for the risky asset is given by  $x_t^n = \frac{E_t^n[s_{t+1}]}{\gamma V[s_{t+1}]}$ . Plugging in our guess for prices, and for  $\beta_0 \neq -1$ , we obtain:

$$x_t^n = \frac{(1 + \beta_0) \theta_t^n + \alpha + \beta_1 d_t + \dots + \beta_q d_{t-q+1} - p_t R}{\gamma (1 + \beta_0)^2 \sigma^2} \quad (33)$$

By market clearing,  $\frac{1}{q} \sum_{n=t-q+1}^t x_t^n = 1$ , which implies that

$$\frac{(1 + \beta_0) \frac{1}{q} \sum_{n=t-q+1}^t \theta_t^n}{\gamma (1 + \beta_0)^2 \sigma^2} + \frac{\alpha + \beta_1 d_t + \dots + \beta_q d_{t-q+1} - p_t R}{\gamma (1 + \beta_0)^2 \sigma^2} = 1.$$

By straightforward algebra and the definition of  $\theta_t^n$ , it follows that

$$(1 + \beta_0) \frac{1}{q} \sum_{n=t-q+1}^t \left[ \sum_{k=0}^{t-n} w(k, \lambda, t-n) d_{t-k} \right] + [\alpha - \gamma (1 + \beta_0)^2 \sigma^2] + \beta_1 d_t + \dots + \beta_q d_{t-q+1} = p_t R.$$

Plugging in (again) our guess for  $p_t$  and using the method of undetermined coefficients, we find the expressions for  $\alpha$  and the  $\beta$ 's:

$$-\frac{\gamma (1 + \beta_0)^2 \sigma^2}{R - 1} = \alpha \quad (34)$$

$$(1 + \beta_0) \frac{1}{q} \sum_{n=t-q+1}^{t-k} w(k, \lambda, t-n) + \beta_{k+1} = \beta_k R \quad \forall k \in \{0, 1, \dots, q-1\} \quad (35)$$

$$0 = \beta_q R. \quad (36)$$

Let  $w_k$  be the average of the weights assigned to dividend  $d_{t-k}$  by each generation in the market at time  $t$ , i.e.,  $w_k = \frac{1}{q} \sum_{n=t-q+1}^t w(k, \lambda, t-n)$ . Given that a weight of zero is assigned to dividends that a generation did not observe, i.e., for  $k > t-n$ , we can rewrite  $w_k = \frac{1}{q} \sum_{n=t-q+1}^{t-k} w(k, \lambda, t-n)$ . Also using  $\beta_q = 0$  from equation (36) we obtain:

$$(1 + \beta_0) w_k + \beta_{k+1} = \beta_k R \quad \forall k \in \{0, 1, \dots, q-2\} \quad (37)$$

$$(1 + \beta_0) w_{q-1} = \beta_{q-1} R \quad (38)$$

By solving this system of equations, we obtain the expressions in the proposition. In particular,  $(1 + \beta_0) (w_{q-2} + w_{q-1}/R) = \beta_{q-2} R$  for  $k = q-2$ ,  $(1 + \beta_0) (w_{q-3} + w_{q-2}/R + w_{q-1}/R^2) = \beta_{q-3} R$  for  $k = q-3$ , and so on. This allow us to express (37) and (38) as

$$(1 + \beta_0) \sum_{j=0}^{k-1} w_{q-(k-j)}/R^j = \beta_{q-k} R \quad \text{for } k = 1, \dots, q. \quad (39)$$

The last expression (39) implies  $\beta_0 = \frac{\sum_{j=0}^{q-1} w_j/R^j}{R - \sum_{j=0}^{q-1} w_j/R^j} = \frac{\sum_{j=0}^{q-1} w_j/R^{j+1}}{1 - \sum_{j=0}^{q-1} w_j/R^{j+1}}$  (from plugging in  $k = q$ ), which in turn, plugged into (34) allows us to obtain the expression for  $\alpha$  from (15) in Proposition 4.2. And expression (39) implies  $\beta_k = \frac{\sum_{j=0}^{q-1-k} w_{k+j}/R^{j+1}}{1 - \sum_{j=0}^{q-1} w_j/R^{j+1}}$  (from substituting  $k$  with  $q-k$ , and using the expression for  $\beta_0$ ) as expressed in equation (16) of the Proposition. The latter also subsumes equation (38), solved for  $\beta_{q-1}$ , and the above formula for  $\beta_0$ , and hence holds for  $k = 0, \dots, q-1$ .  $\square$

*Proof of Proposition 4.3.* For this proof, we will use equations (37) and (38). In addition, note that by construction,  $w_k < w_{k-1}$  for  $\lambda > 0$  since for all generations,  $w(k, \lambda, age)$  is decreasing in  $k$  and more agents observe the realization of  $d_{t-(k-1)}$  than  $d_{t-k}$ . Given this, it follows that since  $\beta_0 > 0$

then  $\beta_{q-1} > 0$  and

$$\beta_{q-1} = \frac{1}{R} (1 + \beta_0) w_{q-1} < \frac{1}{R} [(1 + \beta_0) w_{q-2} + \beta_{q-1}] = \beta_{q-2} \quad (40)$$

In addition, if  $\beta_k < \beta_{k-1}$ , then:

$$\beta_{k-1} = \frac{1}{R} [(1 + \beta_0) w_{k-1} + \beta_k] < \frac{1}{R} [(1 + \beta_0) w_{k-2} + \beta_{k-1}] = \beta_{k-2} \quad (41)$$

Thus, the proof that  $\beta_k < \beta_{k-1}$  for all  $k \in \{1, \dots, q-1\}$  follows by induction.  $\square$

*Proof of Lemma 4.1.* To show that  $\beta_0$  is increasing in  $\lambda$ , let  $G_q(\lambda) = \sum_{k=0}^{q-1} w_k / R^{k+1}$ . We thus have  $\beta_0 = \frac{G_q(\lambda)}{1 - G_q(\lambda)}$ , and it suffices to show that  $G'_q(\lambda) > 0 \quad \forall q > 0$  and  $\forall \lambda > 0$ . After some algebra, the terms in  $G_q(\cdot)$  can be re-organized as follows:

$$G_q(\lambda) = \sum_{age=0}^{q-1} \frac{1}{q} \sum_{k=0}^{age} w(k, \lambda, age) / R^{k+1} \quad (42)$$

Note that for any  $age \in \{0, \dots, q-1\}$ : (i)  $\sum_{k=0}^{age} w(k, \lambda, age) = 1$  and (ii) for any  $\lambda_1, \lambda_2$  such that  $\lambda_1 > \lambda_2 > 0$ ,  $\sum_{k=j}^{age} w(k, \lambda_1, age) < \sum_{k=j}^{age} w(k, \lambda_2, age)$ . Thus, the weight distribution given by  $\lambda_2$  first-order stochastically dominates the weight distribution given by  $\lambda_1$ . Since  $1/R > 1/R^2 > 1/R^3 > \dots > 1/R^{q-1}$ , stochastic dominance implies that for all  $age \in \{0, \dots, q-1\}$ ,  $\sum_{k=0}^{age} c^{k+1} w(k, \lambda_1, age) > \sum_{k=0}^{age} c^{k+1} w(k, \lambda_2, age)$ , and thus  $G_q(\lambda_1) > G_q(\lambda_2)$ .

To show the limit results, note that  $\lim_{\lambda \rightarrow \infty} w(0, \lambda, age) = 1$ , while  $\lim_{\lambda \rightarrow \infty} w(k, \lambda, age) = 0$  for all  $k > 0$ .  $\square$

*Proof of Proposition 4.4.* From Propositions 4.1 and 4.2, we know that, for any  $t$ , any generations  $m \geq n$  both in  $\{t - q + 1, \dots, t\}$  and any  $k \in \{0, \dots, q-1\}$ ,

$$\frac{\partial(x_t^n - x_t^m)}{\partial d_{t-k}} = \frac{(1 + \beta_0)}{\gamma V[s_{t+1}]} \frac{\partial(\theta_t^n - \theta_t^m)}{\partial d_{t-k}}.$$

We note that, for any  $n \in \{t - q + 1, \dots, t\}$ ,  $\frac{\partial \theta_t^n}{\partial d_{t-k}} = w(k, \lambda, n - t)$  if  $k \in \{0, \dots, t - n\}$ , and  $\frac{\partial \theta_t^n}{\partial d_{t-k}} = 0$  if  $k \in \{t - n + 1, \dots, q-1\}$ . (Observe that  $t - n \leq q-1$ .) Hence, it suffices to compare  $w(k, \lambda, t - n)$  with  $w(k, \lambda, t - m)$  for any  $k \in \{0, \dots, q-1\}$ . (As usual, here we adopt the convention that for any  $age$ ,  $w(k, \lambda, age) = 0$  for all  $k \geq age$ .) From Lemma 2.1, there exists a  $k_0$  such that  $w(k, \lambda, t - n) < w(k, \lambda, t - m)$  for all  $k \in \{0, \dots, k_0\}$  and  $w(k, \lambda, t - n) \geq w(k, \lambda, t - m)$  for the rest of the  $k$ 's,  $k \in \{k_0 + 1, \dots, q-1\}$ .  $\square$

The proof of Proposition 4.5 relies on the following first-order stochastic dominance result:

**Lemma B.2.** For any  $a \in \{0, 1, \dots\}$ ,  $a' < a$  and any  $m \in \{0, \dots, a\}$ , let  $F(m, a) \equiv \sum_{j=0}^m w(j, \lambda, a)$ . Suppose the conditions of Lemma 2.1 hold; then  $F(m, a) \leq F(m, a')$  for all  $m \in \{0, \dots, a\}$ .

*Proof.* See Online Appendix OA.1.1.  $\square$

*Proof of Proposition 4.5.* We first introduce some notation. For any  $j \in \{t - n - k + 1, \dots, t - n\}$ , let  $w(j, \lambda, t - n - k) = 0$ ; i.e., we define the weights of generation  $n + k$  for time periods before they were born to be zero. Thus,  $\sum_{j=0}^{t-n-k} w(j, \lambda, t - n - k) d_{t-j} = \sum_{j=0}^{t-n} w(j, \lambda, t - n - k) d_{t-j}$ . In addition, we



note that  $(w(j, \lambda, t - n - k))_{j=0}^{t-n}$  and  $(w(j, \lambda, t - n))_{j=0}^{t-n}$  are sequences of positive weights that add to one.

Let for any  $m \in \{0, \dots, t - n\}$ ,

$$F(m, t - n - k) = \sum_{j=0}^m w(j, \lambda, t - n - k) \text{ and } F(m, t - n) = \sum_{j=0}^m w(j, \lambda, t - n).$$

These quantities, as functions of  $m$ , are non-decreasing and  $F(t - n, t - n - k) = F(t - n, t - n) = 1$ . Moreover,  $F(m + 1, t - n - k) - F(m, t - n - k) = w(m + 1, \lambda, t - n - k)$  and  $F(m + 1, t - n) - F(m, t - n) = w(m + 1, \lambda, t - n)$ . Finally, we set  $F(-1, t - n) = F(-1, t - n - k) = 0$ .

By these observations, by the definition of  $\xi(n, k, t)$ , and by straightforward algebra, it follows that,

$$\begin{aligned} & \xi(n, k, t) \\ &= \frac{\sum_{m=0}^{t-n} (F(m, t - n) - F(m - 1, t - n)) d_{t-m} - \sum_{m=0}^{t-n} (F(m, t - n - k) - F(m - 1, t - n - k)) d_{t-m}}{\gamma(1 + \beta_0)\sigma^2} \\ &= \frac{F(0, t - n) d_t + (F(1, t - n) - F(0, t - n)) d_{t-1} + \dots + (1 - F(t - n - 1, t - n)) d_n}{\gamma(1 + \beta_0)\sigma^2} \\ &\quad - \frac{F(0, t - n - k) d_t + (F(1, t - n - k) - F(0, t - n - k)) d_{t-1} + \dots + (1 - F(t - n - 1, t - n - k)) d_n}{\gamma(1 + \beta_0)\sigma^2} \\ &= \frac{(d_t - d_{t-1}) F(0, t - n) + (d_{t-1} - d_{t-2}) F(1, t - n) + \dots + (d_{n+1} - d_n) F(t - n - 1, t - n) + d_n}{\gamma(1 + \beta_0)\sigma^2} \\ &\quad - \frac{(d_t - d_{t-1}) F(0, t - n - k) + (d_{t-1} - d_{t-2}) F(1, t - n - k) + \dots + (d_{n+1} - d_n) F(t - n - 1, t - n - k) + d_n}{\gamma(1 + \beta_0)\sigma^2} \\ &= \frac{\sum_{j=0}^{t-n-1} (d_{t-j} - d_{t-j-1}) (F(j, t - n) - F(j, t - n - k))}{\gamma(1 + \beta_0)\sigma^2}. \end{aligned}$$

If the weights are non-decreasing, then  $d_{t-j} - d_{t-j-1} \geq 0$  for all  $j = 0, \dots, t - n - 1$ , and it suffices to show that  $F(j, t - n) \leq F(j, t - n - k)$  for all  $j = 0, \dots, t - n - 1$ . This follows from applying Lemma B.2 with  $a = t - n > t - n - k = a'$ .

If the weights are non-increasing, then  $d_{t-j} - d_{t-j-1} \leq 0$ , and the sign of  $\xi(n, k, t)$  changes accordingly.  $\square$

*Proof of Proposition 4.6.* By Propositions 4.1 and 4.2, it follows that for any  $t$  and  $n \leq t$ ,

$$\begin{aligned} x_t^n &= \frac{1}{\gamma\sigma^2(1 + \beta_0)^2} \left( \alpha_0 + (1 + \beta_0)\theta_t^n + \sum_{k=1}^{q-1} \beta_k d_{t+1-k} - R \left( \alpha_0 + \sum_{k=0}^{q-1} \beta_k d_{t-k} \right) \right) \\ &= \frac{1}{\gamma\sigma^2(1 + \beta_0)^2} \left( \alpha_0(1 - R) + (1 + \beta_0)\theta_t^n - R\beta_0 d_t + \sum_{k=1}^{q-1} \beta_k (d_{t+1-k} - R d_{t-k}) \right). \end{aligned} \quad (43)$$

Thus, for  $n \in \{t - q + 1, \dots, t - 1\}$ ,

$$x_t^n - x_{t-1}^n = \frac{(1 + \beta_0)(\theta_t^n - \theta_{t-1}^n) + \mathcal{T}(d_{t:t-q})}{\gamma\sigma^2(1 + \beta_0)^2} \quad (44)$$

where  $\mathcal{T}(d_{t:t-q}) \equiv \sum_{k=1}^{q-1} \beta_k (d_{t+1-k} - d_{t-k} - R(d_{t-k} - d_{t-1-k})) - R\beta_0(d_t - d_{t-1})$ . Note that  $\mathcal{T}(d_{t:t-q})$  is not cohort specific, i.e., does not depend on  $n$ .

The fact that  $x_t^t - x_{t-1}^t = x_t^t$  and  $x_t^{t-q} - x_{t-1}^{t-q} = -x_{t-1}^{t-q}$ , and market clearing imply  $q^{-1} \left( \sum_{n=t-q}^t x_t^n - x_{t-1}^n \right) = 0$ . This expression and the expression in (44) imply that

$$\frac{1}{q} \left( \sum_{n=t-q+1}^{t-1} \frac{(1+\beta_0)(\theta_t^n - \theta_{t-1}^n)}{\gamma\sigma^2(1+\beta_0)^2} + x_t^t - x_{t-1}^{t-q} \right) = -\frac{1}{q} \sum_{n=t-q}^t \frac{\mathcal{T}(d_{t:t-q})}{\gamma\sigma^2(1+\beta_0)^2} = -\frac{\mathcal{T}(d_{t:t-q})}{\gamma\sigma^2(1+\beta_0)^2}.$$

Letting  $\theta_{t-1}^t = \theta_t^{t-q} = 0$  and applying (43) to  $x_t^t$  and  $x_t^{t-q}$ , it follows that

$$\frac{1}{q} \left( \sum_{n=t-q}^t (1+\beta_0)(\theta_t^n - \theta_{t-1}^n) \right) = -\mathcal{T}(d_{t:t-q}).$$

Thus, we can express the change in individual demands in expression (44) as follows:

$$x_t^n - x_{t-1}^n = \chi \left[ (\theta_t^n - \theta_{t-1}^n) - \frac{1}{q} \sum_{n=t-q}^t (\theta_t^n - \theta_{t-1}^n) \right], \quad \forall n \in \{t, \dots, t-q\}$$

where  $\chi \equiv \frac{1}{\gamma\sigma^2(1+\beta_0)}$ . By squaring and summing at both sides the desired result follows.  $\square$

## Appendix C Proofs for Results in Section 7

To establish the results in this section we need the following lemmas (the proofs are relegated to the Online Appendix OA.1.2).

**Lemma C.1.** Suppose  $z \sim N(\mu, \sigma^2)$ , then for any  $A, B \in \mathbb{R}$  and  $C \geq 0$ ,  $z \mapsto K^{-1} \exp\{-A - Bz - Cz^2\} \phi(z; \mu, \sigma^2)$  is Gaussian with mean  $m \equiv -\Sigma^2 B + \Sigma^2 \sigma^{-2} \mu$  and  $\Sigma^2 \equiv \frac{\sigma^2}{2C\sigma^2 + 1}$ , where

$$K = E_{N(\mu, \sigma^2)}[\exp\{-A - Bz - Cz^2\}] = \frac{1}{\sqrt{2\sigma^2 C + 1}} \exp\{-(A + 0.5\sigma^{-2}\mu^2) + \frac{m^2}{2\Sigma^2}\}$$

**Lemma C.2.** Demands for the risky asset in the last two period of an agent's life are given by:  $x_{t+q}^t = 0$  and  $x_{t+q-1}^t = \frac{E_{t+q-1}^t[s_{t+q}]}{\gamma\sigma^2(1+\beta_0)^2}$ ,  $\forall t \in \mathbb{Z}, q \geq 1$ .

**Lemma C.3.** Let  $z \sim N(\mu, \sigma^2)$ . Let  $A, B \in \mathbb{R}$  and  $C \geq 0$ , and  $z \mapsto h(z) \equiv f + ez$  for any  $e, f \in \mathbb{R}$ . Then

$$\begin{aligned} \max_x E[-\exp\{-A - Bz - Cz^2\} \exp\{-axh(z)\}] &= -\frac{1}{\sqrt{2\sigma^2 C + 1}} \exp\left[-A - \frac{1}{2} \left( \frac{\mu^2}{\sigma^2} - \frac{m^2}{s^2} \right)\right] \exp\left[-\frac{1}{2} \frac{\tilde{\mu}(m, s^2)^2}{\tilde{\sigma}^2(m, s^2)}\right] \\ \arg \max_x E[-\exp\{-A - Bz - Cz^2\} \exp\{-axh(z)\}] &= \frac{\tilde{\mu}(m, s^2)}{a\tilde{\sigma}^2(m, s^2)} \end{aligned}$$

with  $m = s^2 [\sigma^{-2}\mu - B]$ ,  $s^2 = \frac{\sigma^2}{2C\sigma^2 + 1}$ ,  $\tilde{\mu}(m, s^2) = E_{N(m, s^2)}[h(z)]$ ,  $\sigma^2(m, s^2) = V_{N(m, s^2)}[h(z)]$ .

Let  $\beta(k) = \beta_{k+1} - r\beta_k$  for  $k \in \{0, \dots, K-1\}$  and  $\beta(K) = -r\beta_K$ .

**Lemma C.4.** Suppose  $p_t = \alpha + \sum_{k=0}^K \beta_k d_{t-k}$  with  $\beta_0 \neq -1$ . Then the demand for risky assets of any cohort alive at time  $t$  is an affine function of past dividends, where the coefficients associated with a given dividend will depend on the agent's age,  $age$ . That is,

$$x_t^{t-age} = \delta(age) + \sum_{k=0}^K \delta_k(age) d_{t-k}, \text{ for } age \in \{0, \dots, q\} \quad (45)$$

with

$$\delta(q) = \delta_k(q) = 0, \quad \forall k \in \{0, \dots, K\} \quad (46)$$

$$\delta(q-1) = \frac{\alpha(1-R)}{\gamma((1+\beta_0)\sigma)^2}, \quad \delta_k(q-1) = \frac{(1+\beta_0)w(k, \lambda, q-1) + \beta(k)}{\gamma((1+\beta_0)\sigma)^2}, \quad \forall k \in \{0, \dots, q-1\} \quad (47)$$

$$\delta_k(q-1) = \frac{\beta(k)}{\gamma((1+\beta_0)\sigma)^2}, \quad \forall k \in \{q, \dots, K\}, \quad (48)$$

and for  $age \in \{0, \dots, q-2\}$ ,

$$\delta(age) = \frac{\alpha(1-R) - s_{age}^2(1+\beta_0)\delta_0(age+1)\delta(age+1)(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}, \quad (49)$$

$$\delta_k(age) = \frac{(1+\beta_0)s_{age}^2(\sigma^{-2}w(k, \lambda, age) - [(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2\delta_{k+1}(age+1)\delta_0(age+1)]) + \beta(k)}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2} \quad (50)$$

$$\forall k \in \{0, \dots, q-1\},$$

$$\delta_k(age) = \frac{-(1+\beta_0)s_{age}^2[(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2\delta_{k+1}(age+1)\delta_0(age+1)] + \beta(k)}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}, \quad (51)$$

$$\forall k \in \{q, \dots, K-1\}$$

$$\delta_K(age) = \frac{\beta(K)}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}, \quad (52)$$

$$\text{and } s_{q-1} = \sigma \text{ and } s_{age}^2 \equiv \frac{\sigma^2}{(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2(\delta_0(age+1))^2\sigma^2+1}$$

We now prove Proposition 7.1. We first note that the expressions for  $b_j, b_j(k)$  and  $c_j$  for  $j \in \{0, \dots, q-1\}$  in the Proposition are:

$$b_j \equiv (R^{q-1-j}\gamma)^2((1+\beta_0)\sigma_j)^2\delta(j)\delta_0(j)$$

$$b_j(k) \equiv \delta_k(j)\delta_0(j)(R^{q-1-j}\gamma)^2((1+\beta_0)\sigma_j)^2$$

and,  $c_{q-1} = 1$  and

$$c_{j-1} = 0.5(R^{q-1-(j+1)}\gamma)(1+\beta_0)\sigma_{j+1}\delta_0(j+1)$$

for  $j \in \{0, \dots, q-2\}$ .

*Proof of Proposition 7.1.* By lemma B.1,  $x_{t+q-1}^t = \frac{E_{N(m_{q-1}, \sigma_{q-1}^2)}^{[s_{t+q}]}}{\gamma V_{N(m_{q-1}, \sigma_{q-1}^2)}^{[s_{t+q}]}}$ , with  $m_{q-1} = \theta_{t+q-1}^t$

and  $\sigma_{q-1} = \sigma$ , and

$$V_{t+q-1}^t = -\exp\{-0.5((1+\beta_0)\sigma\gamma x_{t+q-1}^t)^2\}.$$

By lemma C.4,  $x_{t+q-1}^t$  is affine in  $d_{t+q-1-K:t_q}$  and thus  $V_{t+q-1}^t = -\exp\{-A - Bd_{t+q-1} - C(d_{t+q-1})^2\}$  where  $A$ ,  $B$  and  $C$  depend on primitives and on  $d_{t+q-1-K:t+q-2}$ , in particular  $B$  is affine in  $d_{t+q-1-K:t+q-1-1}$  and  $C$  is constant with respect to  $d_{t+q-1-K:t+q-1}$ :

$$\begin{aligned} C &\equiv \frac{1}{2}\gamma^2((1+\beta_0)\sigma_{q-1})^2(\delta_0(q-1))^2 \\ B &\equiv \gamma^2((1+\beta_0)\sigma_{q-1})^2 \left( \delta(q-1) + \sum_{j=1}^K \delta_k(q-1)d_{t+q-1-j} \right) \delta_0(q-1) \\ A &\equiv \frac{1}{2}\gamma^2((1+\beta_0)\sigma_{q-1})^2 \left( \delta(q-1) + \sum_{j=1}^K \delta_k(q-1)d_{t+q-1-j} \right)^2. \end{aligned}$$

(see Lemma C.4 for the expressions for  $\delta(q-1)$  and  $(\delta_k(q-1))_{k=1}^K$ ).

At time  $t+q-2$ , by equation (25),

$$x_{t+q-2}^t = \arg \max_{x \in \mathbb{R}} E_{t+q-2}^t [V_{t+q-1}^t(d_{t+q-1-K:t+q-1}) \exp(-\gamma s_{t+q-1}x)]$$

where the expectation is taken with respect  $N(\theta_{t+q-2}^t, \sigma^2)$ . Hence, by lemma C.1, this problem can be cast as

$$x_{t+q-2}^t = \arg \max_{x \in \mathbb{R}} E_{N(m_{q-2}, \sigma_{q-2}^2)} [-\exp(-R\gamma s_{t+q-1}x)]$$

where  $m_{q-2} = \sigma_{q-2}(\frac{\theta_{t+q-2}^t}{\sigma^2} - B)$  and  $\sigma_{q-2}^2 = \frac{\sigma^2}{2C\sigma^2+1}$ . Hence, by lemma B.1

$$x_{t+q-2}^t = \frac{E_{N(m_{q-2}, \sigma_{q-2}^2)}[s_{t+q-1}]}{\gamma R V_{N(m_{q-2}, \sigma_{q-2}^2)}[s_{t+q-1}]},$$

Also, by lemma B.1,  $V_{t+q-2}^t = -\exp\{-0.5(V_{N(m_{q-2}, \sigma_{q-2}^2)}[s_{t+q-1}]R\gamma x_{t+q-2}^t)^2\}$ . By lemma C.4,  $x_{t+q-2}^t$  is affine and thus  $V_{t+q-2}^t = -\exp\{-A - Bd_{t+q-2} - C(d_{t+q-2})^2\}$  where  $A$ ,  $B$  and  $C$  depend on primitives and on  $d_{t+q-2-K:t+q-3}$ , in particular  $B$  is affine in  $d_{t+q-2-K:t+q-3}$

and  $C$  is constant with respect to  $d_{t+q-1-K:t+q-1}$ :

$$\begin{aligned} C &\equiv \frac{1}{2}(R\gamma)^2((1+\beta_0)\sigma_{q-2})^2(\delta_0(q-2))^2 \\ B &\equiv (R\gamma)^2((1+\beta_0)\sigma_{q-2})^2 \left( \delta(q-2) + \sum_{j=1}^K \delta_k(q-2)d_{t+q-2-j} \right) \delta_0(q-2) \\ A &\equiv \frac{1}{2}(R\gamma)^2((1+\beta_0)\sigma_{q-2})^2 \left( \delta(q-2) + \sum_{j=1}^K \delta_k(q-2)d_{t+q-2-j} \right)^2. \end{aligned}$$

(observe that the  $A$  and  $B$  and  $C$  are not the same as the previous ones; the expressions for  $\delta(q-2)$  and  $(\delta_k(q-2))_{k=1}^K$  can be found in the statement of lemma C.4).

The result for  $j \in \{0, \dots, q-3\}$  follows by iteration.  $\square$

*Proof of Proposition 7.2.* Market Clearing and Lemma C.4 imply that, for all  $k \in \{0, \dots, K\}$ ,  $\sum_{age=0}^{q-1} \delta_k(age) = 0$  and  $\sum_{age=0}^{q-1} \delta(age) = q$ .

For  $k = K$ , it follows from equations 48 and 52

$$\sum_{age=0}^{q-1} \delta_K(age) = \beta(K) \left( \sum_{age=0}^{q-1} \frac{1}{R^{q-1-age}\gamma((1+\beta_0)s_{age})^2} + \frac{1}{\gamma((1+\beta_0)\sigma)^2} \right)$$

therefore  $\beta(K) = 0$  which implies that  $\beta_K = 0$  and  $\beta(K-1) = -R\beta_{K-1}$  and  $\delta_K(age) = 0$  for any  $age$ .

For  $k = K-1$ , by equations 48 and 51

$$\sum_{age=0}^{q-1} \delta_{K-1}(age) = \beta(K-1) \left( \sum_{age=0}^{q-2} \frac{1}{R^{q-1}\gamma((1+\beta_0)s_{age})^2} + \frac{1}{\gamma((1+\beta_0)\sigma)^2} \right)$$

and thus  $\beta(K-1) = 0$  which implies that  $\beta_{K-1} = 0$  and  $\beta(K-2) = -R\beta_{K-2}$  and  $\delta_{K-1}(age) = 0$  for any  $age$ .

By induction, for any  $k \in \{q, \dots, K-2\}$ , taking  $\beta_{k+1} = 0$ , it follows by equations 48 and 51, that

$$\sum_{age=0}^{q-1} \delta_k(age) = \beta(k) \left( \sum_{age=0}^{q-2} \frac{1}{R^{q-1-age}\gamma((1+\beta_0)s_{age})^2} + \frac{1}{\gamma((1+\beta_0)\sigma)^2} \right)$$

and thus  $\beta(k) = 0$  which implies  $\beta_k = 0$  and  $\beta(k-1) = -R\beta_{k-1}$  and  $\delta_k(age) = 0$  for any  $age \in \{q, \dots, K\}$ .  $\square$

*Proof of Proposition 7.3.* Throughout the proof, let  $w_0 \equiv w(0, \lambda, 0)$ .

We know from Lemma OA.2.1 in the Online Appendix OA.2 that  $\{\alpha, \beta_0, \beta_1\}$  solve the system of equations given by (73) and (74) and (72) in Online Appendix OA.2.

STEP 1. By equation (72),

$$2R\gamma(1+\beta_0)^2\sigma^2 = \alpha(1-R) \left[ R + \frac{\sigma^2}{s^2} - \frac{[(1+\beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0]}{1+\beta_0} \right].$$

We note that  $R > 1 \geq w(0, \lambda, 0)$ , thus, if  $0 < \beta_1 < R\beta_0$  and  $1 + \beta_0 > 0$ , then

$$\left[ R + \frac{\sigma^2}{s^2} - \frac{[(1+\beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0]}{1+\beta_0} \right] > 0$$

and  $\alpha \leq 0$ .

STEP 2. We show that if  $1 + \beta_0 > 0$ , then  $0 < \beta_1 < R\beta_0$ .

For  $1 + \beta_0 > 0$ , equation (74) implies  $\beta_1 > 0$  and  $l(1, 1) > 0$ . Now assume that  $\beta_1 - R\beta_0 > 0$ , this implies that  $l(0, 1) > 0$ . For equation (73) to hold it must be that  $1 - \frac{1}{R} \frac{l(1, 1)}{1 + \beta_0} < 0$ .

$$1 - \frac{1}{R} \frac{l(1, 1)}{(1 + \beta_0)^2} = 1 - \frac{1}{R} (1 - w_0) + \frac{\beta_1}{1 + \beta_0} > 0 \quad (53)$$

Since  $R > 1$ ,  $w_0 < 1$ , and  $\beta_1 > 1$ . Contradiction. Then,  $1 + \beta_0 > 0 \Rightarrow \beta_1 - R\beta_0 < 0$ .

STEP 3. We now show that  $1 + \beta_0 > 0$ . Let  $\phi \equiv \frac{\sigma^2}{s^2} > 1$ . From equation (74):

$$\frac{(1 + \beta_0)(1 - w_0)}{\phi + R} = \beta_1.$$

We plug this into equation (73) and we obtain:

$$\begin{aligned} & \phi \left( -\beta_0 R + \frac{(1 + \beta_0)(1 - w_0)}{\phi + R} \right) + R \left[ \frac{(1 + \beta_0)(1 - w_0)}{\phi + r} + (1 + \beta_0)w_0 - \beta_0 R \right] + \dots \\ & + \left[ 1 + \beta_0 - \frac{\phi(1 - w_0)(1 + \beta_0 - \beta_0 \phi R - \beta_0 R^2 + (1 + \beta_0)(\phi + R - 1)w_0)}{(\phi + R)^2} \right] = 0. \end{aligned}$$

Note that this is a linear equation on  $\beta_0$ . Therefore,

$$\beta_0 = - \frac{2 - w_0(1 - R) - \frac{\phi(1 - w_0)(1 + (\phi + R - 1)w_0)}{(\phi + R)^2}}{2 - w_0(1 - R) - \frac{\phi(1 - w_0)(1 + (\phi + R - 1)w_0)}{(\phi + R)^2} - (R\phi + R^2) \left[ 1 - \frac{\phi(1 - w_0)}{(\phi + R)^2} \right]} \equiv - \frac{A}{A - x}.$$

where  $A \equiv 2 - w_0(1 - R) - \frac{\phi(1 - w_0)(1 + (\phi + R - 1)w_0)}{(\phi + R)^2}$  and  $x \equiv (R\phi + R^2) \left[ 1 - \frac{\phi(1 - w_0)}{(\phi + R)^2} \right] > 0$ .

Note that for  $x = 0 \Rightarrow \beta_0 = -1$ . Then, it suffices to show that  $\frac{\partial \beta_0}{\partial x} = \frac{A}{(A - x)^2} \geq 0$ , that is,  $A \geq 0$ . For  $w_0 = 0.5$ , which corresponds to  $\lambda = 0$ ,  $A$  is positive, i.e.,  $A(0.5) > 0$ . In addition,  $\frac{\partial A}{\partial w_0} = \frac{(\phi + R - 1)(R^2 + \phi(R - 2(1 - w_0)))}{(\phi + R)^2} > 0$  for  $w_0 \geq 0.5$ . Therefore,  $A > 0$  for  $w_0 \geq 0.5$ .

If we are interested in  $\lambda < 0$  cases, since  $A(0) > 0$ , all we need to ensure that  $A$  is positive, and thus the result holds for  $w_0 \in [0, 0.5)$ , is that  $R \geq 2(1 - w_0)$ .  $\square$

In order to show Proposition 7.4, we need the following Lemmas (their proofs are relegated to the Online Appendix OA.1.2).

**Lemma C.5.** For  $\lambda \geq 0, 1 + \beta_0 + \beta_1 - r\beta_0 > 0$ .

**Lemma C.6.** Given our linear guess for prices (7), when  $q = 2$ , at time  $t$ :

$$x_t^{t-1} = \frac{E_t^{t-1}[s_{t+1}]}{\gamma R(1 + \beta_0)\sigma^2} = \frac{\alpha(1 - R)}{\gamma(1 + \beta_0)^2\sigma^2} + \frac{l(0, 1)}{\gamma(1 + \beta_0)^2\sigma^2}d_t + \frac{l(1, 1)}{\gamma(1 + \beta_0)^2\sigma^2}d_{t-1} \quad (54)$$

$$x_t^t = \frac{E_{\Phi(m, s^2)}[s_{t+1}]}{R(1 + \beta_0)s^2} = \delta(0) + \delta_0(0)d_t + \delta_1(0)d_{t-1} \quad (55)$$

with  $l(0, 1) \equiv [(1 + \beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0]$  and  $l(1, 1) \equiv [(1 + \beta_0)w(1, \lambda, 0) - R\beta_1]$ , and  $\delta(0) = \frac{\alpha(1-R)[1 - \frac{s^2}{\sigma^2} \frac{l(0,1)}{(1+\beta_0)}]}{\gamma R(1+\beta_0)^2 s^2}$ ,  $\delta_0(0) = \frac{\beta_1 - R\beta_0 + (1+\beta_0) \frac{s^2}{\sigma^2} \left(1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2}\right)}{\gamma R(1+\beta_0)^2 s^2}$ , and  $\delta_1(0) = -\frac{R\beta_1}{R\gamma(1+\beta_0)s^2}$ .

*Proof of Proposition 7.4.* By lemma C.6 and Market Clearing, it follows that

$$\delta_0(0) + \frac{l(0, 1)}{\gamma(1 + \beta_0)^2\sigma^2} = 0,$$

and

$$\delta_1(0) + \frac{l(1, 1)}{\gamma(1 + \beta_0)^2\sigma^2} = 0.$$

These expressions and Lemma C.6 imply that  $\frac{\partial x_t^{t-1}}{\partial d_t} = \frac{l(0,1)}{\gamma(1+\beta_0)^2\sigma^2} = -\frac{\partial x_t^t}{\partial d_t}$ , and  $\frac{\partial x_t^t}{\partial d_{t-1}} = \delta_1(0) = -\frac{\partial x_t^t}{\partial d_{t-1}}$ . Therefore, it suffices to show that  $l(0, 1) < 0$  and  $\delta_1(0) < 0$ .

By proposition 7.3,  $\beta_1 > 0$  and  $\beta_0 > 0$  and thus  $\delta_1(0) = -\frac{R\beta_1}{R\gamma(1+\beta_0)s^2} < 0$ . So it only remains to show that  $l(0, 1) < 0$ . To show this, note that from the equilibrium condition (73) we have:

$$0 = \left[ R - \frac{l(1, 1)}{(1 + \beta_0)} \right] l(0, 1) + \frac{l(0, 1)^2}{(1 + \beta_0)^2} (\beta_1 - R\beta_0) + [1 + \beta_0 + \beta_1 - R\beta_0]$$

From Lemma C.5,  $1 + \beta_0 + \beta_1 - R\beta_0 > 0$ . Let  $x = \frac{l(0,1)}{1+\beta_0}$ , then

$$0 = [R(1 + \beta_0) - l(1, 1)]x + x^2(\beta_1 - R\beta_0) + [1 + \beta_0 + \beta_1 - R\beta_0]$$

or equivalently  $F(x) \equiv ax^2 + bx + c$ , with  $a = \beta_1 - R\beta_0 < 0$  (by Proposition 7.3),  $b = R(1 + \beta_0) - l(1, 1) = R(1 + \beta_0) - (1 + \beta_0)w(1, \lambda, 0) + R\beta_1 > 0$  (by Proposition 7.3) and  $c = 1 + \beta_0 + \beta_1 - R\beta_0 > 0$  (by Lemma C.5). Thus:  $F$  is concave and  $F(0) = c > 0$ . By definition of  $F$ ,  $l(0, 1)/(1 + \beta_0)$  must be a root of  $F$ . Let  $x^* = \arg \max_{x \in \mathbb{R}} F(x)$ , which is given by  $x^* = -\frac{b}{2a} > 0$ . Therefore,  $F(\cdot)$  has two roots  $x_1, x_2$  with  $x_1 < 0 < x^* < x_2$ .

We now show that  $x_2 = \frac{l(0,1)}{1+\beta_0}$  cannot hold. Suppose not, that is assume that our solution is the positive root  $\frac{l(0,1)}{1+\beta_0} = x_2$ , then since  $x^* < x_2$  and  $a < 0$ :  $\frac{b}{2} < -a \frac{l(0,1)}{1+\beta_0}$ , or equivalently,  $\frac{R(1+\beta_0)-l(1,1)}{2} < l(0,1) \frac{R\beta_0-\beta_1}{1+\beta_0}$ .

Let  $Z \equiv -\frac{\beta_1-R\beta_0}{1+\beta_0}$ , then the last inequality implies that  $R(1+\beta_0) - (1+\beta_0)(1-w_0) + R\beta_1 < 2l(0,1)Z$  (recall that  $w_0 \equiv w(0, \lambda, 0)$  and  $w(1, \lambda, 0) = 1 - w_0$ ). By replacing  $l(0,1)$  and some algebra, it follows that  $R(1+\beta_0) - (1+\beta_0)(1-w_0) + R\beta_1 < 2Z[(1+\beta_0)w_0 + \beta_1 - R\beta_0]$  or equivalently  $\frac{1}{2}w_0 + \frac{1}{2}\left[R - 1 + R\frac{\beta_1}{1+\beta_0}\right] < Z(w_0 - Z)$ . We note that the RHS is bounded by  $w_0/4$  since no matter the value of  $Z$ , the function  $z \mapsto z(w_0 - z)$  cannot be larger than  $(w_0)^2/4 < w_0/4$  (since  $w_0 \in [0, 1]$ ). Therefore,

$$\frac{w_0}{4} > \frac{w_0}{2} + \frac{1}{2}\left[R - 1 + R\frac{\beta_1}{1+\beta_0}\right].$$

However,  $\frac{1}{2}\left[R - 1 + R\frac{\beta_1}{1+\beta_0}\right] > 0$ ; thus a contradiction follows. The solution must be the negative root.  $\square$



# Online Appendix

## Appendix OA.1 Proofs of Supplementary Lemmas

### OA.1.1 Proofs of Supplementary Lemmas in the Appendix B

*Proof of Lemma B.1.* Since  $z \sim N(\mu, \sigma^2)$ , we can re-write the problem as follows:

$$\begin{aligned} x^* &= \arg \max_x -\exp\left(-axE[z] + \frac{1}{2}a^2x^2V[z]\right) \\ &= \arg \max_x ax\mu - \frac{1}{2}a^2x^2\sigma^2 \end{aligned}$$

From FOC,  $x^* = \frac{\mu}{a\sigma^2}$ . Plugging  $x^*$  into  $-\exp(-ax^*\mu + \frac{1}{2}a^2(x^*)^2\sigma^2)$  the second result follows.  $\square$

*Proof of Lemma B.2.* From Lemma 2.1, we know that there exists a unique  $j_0$  where  $w(j_0, \lambda, a') - w(j_0, \lambda, a)$  “crosses” zero. Thus, for  $m \leq j_0$ , the result is true because  $w(j, \lambda, a') > w(j, \lambda, a)$  for all  $j \in \{0, \dots, m\}$ . For  $m > j_0$ , the result follows from the fact that  $w(j, \lambda, a') < w(j, \lambda, a)$  for all  $j \in \{m, \dots, a\}$  and  $F(a, a) = F(a', a') = 1$ .  $\square$

### OA.1.2 Proof of Supplementary Lemmas in the Appendix C

*Proof of Lemma C.1.* Let  $\varphi(z) \equiv K \exp\{-(A + Bz + Cz^2)\}\phi(z; \mu, \sigma^2)$ . By definition of  $K$ ,  $\int \varphi(z)dz = 1$  and  $\varphi \geq 0$ , so it is a pdf. Moreover,

$$\begin{aligned} \varphi(z) &= \frac{K^{-1}}{\sqrt{2\pi}\sigma} \exp\{-A - Bz - Cz^2 - 0.5\sigma^{-2}(z - \mu)^2\} \\ &= \frac{1}{K\sqrt{2\pi}\sigma} \exp\{-z^2(C + 0.5\sigma^{-2}) - 2z(0.5B - 0.5\sigma^{-2}\mu) - (A + 0.5\sigma^{-2}\mu^2)\} \\ &= \frac{1}{K\sqrt{2\pi}\sigma} \exp\{-(A + 0.5\sigma^{-2}\mu^2)\} \exp\{-0.5(2C + \sigma^{-2})\left(z^2 - 2z\frac{(-B + \sigma^{-2}\mu)}{(2C + \sigma^{-2})}\right)\}. \end{aligned}$$

Let  $\Sigma^2 \equiv (2c + \sigma^{-2})^{-1}$ ,  $m \equiv \Sigma^2(\sigma^{-2}\mu - b)$ , and  $K = \frac{1}{\sqrt{2\sigma^2C+1}} \exp\{-(A + 0.5\sigma^{-2}\mu^2) + \frac{m^2}{2\Sigma^2}\}$ :

$$\begin{aligned} \varphi(z) &= \frac{1}{K\sqrt{2\pi}\sigma} \exp\{-(a + 0.5\frac{\mu^2}{\sigma^2}) + \frac{m^2}{2\Sigma^2}\} \exp\{-\frac{z^2 - 2zm + m^2}{2\Sigma^2}\} \\ &= \frac{1}{K\sqrt{2\pi}\sigma} \exp\{-(a + 0.5\sigma^{-2}\mu^2) + \frac{m^2}{2\Sigma^2}\} \exp\{-\frac{(z - m)^2}{2\Sigma^2}\} = \frac{1}{\sqrt{2\pi}\Sigma} \exp\{-\frac{(z - m)^2}{2\Sigma^2}\} \\ &= \frac{1}{\sqrt{2\pi}\Sigma^2} \exp\{-\frac{(z - m)^2}{2\Sigma^2}\} \end{aligned}$$

$\square$

*Proof of Lemma C.2.* At time  $t + q$ , an agent born in  $t$  is in the last period of his life, consuming all of its wealth. Therefore, he will sell all of its claims to the assets it holds and consume. The gain from saving is zero, and therefore the holding of financial assets is also zero by the end of this period:  $x_{t+q}^t = 0, a_{t+q}^t = 0$ . Given this, we can compute the portfolio choice of an agent with age  $q - 1$ , who does want to save for next period when all wealth will be consumed. The agent's problem is a standard static portfolio problem, with initial wealth  $W_{tq}^t$ :

$$\max_x E_{t+q-1}^t [-\exp(-\gamma(W_{t+q-1}^t + xs_{t+q}))] = \max_x E_{t+q-1}^t [-\exp(-\gamma xs_{t+q})] \quad (56)$$

At time  $t + q - 1$ , the only random variable is  $d_{t+q}$ , which is normally distributed, and thus  $s_{t+q} \sim N(E_{t+q-1}^t[s_{t+q}]; (1 + \beta_0)\sigma^2)$ . Given this, the agent's problem becomes:

$$\max_x \left[ -\exp\left(-\gamma x E_{t+q-1}^t[s_{t+q}] + \frac{1}{2}\gamma^2 x^2 (1 + \beta_0)\sigma^2\right) \right] \quad (57)$$

$$\iff \max_x x E_{t+q-1}^t[s_{t+q}] - \frac{1}{2}\gamma x^2 (1 + \beta_0)^2 \sigma^2. \quad (58)$$

And therefore, by FOC:

$$x_{t+q-1}^t = \frac{E_{t+q-1}^t[s_{t+q}]}{\gamma\sigma^2(1 + \beta_0)^2}. \quad (59)$$

□

*Proof of Lemma C.3.* Note that  $E[-\exp\{-A - Bz - Cz^2\}\exp\{-axh(z)\}]$  can be written as:

$$\int \exp\{-axh(z)\} - \exp\{-A - Bz - Cz^2\} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\frac{z - \mu}{\sigma^2}\right\} dz$$

By Lemma C.1, we know that his can be re-written as:

$$\frac{1}{\sqrt{2\sigma^2 C + 1}} \exp\left\{-A - 0.5\left(\frac{\mu^2}{\sigma^2} - \frac{m^2}{s^2}\right)\right\} \int -\exp\{-axh(z)\} \Phi(m, s^2) dz$$

with  $m = -s^2 B + s\sigma^{-2}\mu$  and  $s^2 = \frac{\sigma^2}{2C\sigma^2 + 1}$ . Therefore, the maximization problem becomes:

$$\max_x E_{N(m, s^2)}[-\exp\{-axh(z)\}]$$

with  $E_{N(m, s^2)}[\cdot]$  being the expectations operator over  $z \sim N(m, s^2)$ . Since  $h(z)$  is linear, we know that  $h(z) \sim N(\tilde{\mu}(m, s^2), \tilde{\sigma}(m, s^2)^2)$ , with  $\tilde{\mu}(m, s^2) = E_{N(m, s^2)}[h(z)]$ ,  $\tilde{\sigma}(m, s^2)^2 =$

$V_{N(m,s^2)}[h(z)]$ , by Lemma B.1, we know that

$$\begin{aligned} \arg \max_x E[-\exp\{-A - Bz - Cz^2\} \exp\{-axh(z)\}] &= \frac{\tilde{\mu}(m, s^2)}{a\tilde{\sigma}(m, s^2)^2} \\ \max_x E[-\exp\{-A - Bz - Cz^2\} \exp\{-axh(z)\}] &= -\frac{1}{\sqrt{2\sigma^2 C + 1}} \exp\left[-A - 0.5\left(\frac{\mu^2}{\sigma^2} - \frac{m^2}{s^2}\right)\right] \\ &\quad \times \exp\left[-0.5\frac{\tilde{\mu}(m, s^2)^2}{\tilde{\sigma}(m, s^2)^2}\right] \end{aligned}$$

□

Let  $t \mapsto \rho(t) \equiv \gamma t^2$  and let

$$\begin{aligned} \Lambda(d_{t-K}, \dots, d_t) &\equiv \alpha(1 - R) + \sum_{k=1}^K \beta_k d_{t+1-k} - R \sum_{k=0}^K \beta_k d_{t-k} \\ &= \alpha(1 - R) + \sum_{j=0}^{K-1} \beta_{j+1} d_{t-j} - R \sum_{k=0}^K \beta_k d_{t-k} = \alpha(1 - R) + \sum_{k=0}^K \beta(k) d_{t-k} \end{aligned}$$

with  $\beta(k) = \beta_{k+1} - R\beta_k$  for  $k \in \{0, \dots, K-1\}$  and  $\beta(K) = -R\beta_K$ . We use  $\Lambda_\tau$  to denote  $\Lambda(d_{\tau-K}, \dots, d_\tau)$ .

*Proof of Lemma C.4.* We divide the proof into several steps.

STEP 1 It is straightforward that demand for risky assets can only be positive for a generation that is alive. From Lemma C.2, we know that  $x_t^{t-q} = 0$  and that  $x_t^{t-q+1} = \frac{E_t^{t-q+1}[s_{t+1}]}{\gamma((1+\beta_0)\sigma)^2}$ . Therefore,

$$\delta(q) = \delta_k(q) = 0, \quad \forall k \in \{0, \dots, K\} \quad (60)$$

$$\delta(q-1) = \frac{\alpha(1-R)}{\gamma((1+\beta_0)\sigma)^2}, \quad \delta_k(q-1) = \frac{(1+\beta_0)w(k, \lambda, q-1) + \beta(k)}{\gamma((1+\beta_0)\sigma)^2}, \quad \forall k \in \{0, \dots, q-1\} \quad (61)$$

$$\delta_k(q-1) = \frac{\beta(k)}{\gamma((1+\beta_0)\sigma)^2}, \quad \forall k \in \{q, \dots, K\}. \quad (62)$$

We also know from Lemma B.1 that

$$V^{q-1}(d_{t-K}, \dots, d_t) = -\exp\left(-\frac{1}{2}\left(d_t \delta_0(q-1) + \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t-j}\right)^2 \gamma^2((1+\beta_0)s_{q-1})^2\right)$$

where  $s_{q-1} = \sigma^2$ . Henceforth, we denote  $V^{q-1}(d_{t-K}, \dots, d_t)$  by  $V_t^{t-q+1}$ . In particular,  $V_{t+1}^{t+1-q+1} = V_{t+1}^{t-q+2} = V^{q-1}(d_{t+1-K}, \dots, d_{t+1})$ .

STEP 2. We now derive the risky demand and continuation value for generation aged  $q-2$ . The problem of generation aged  $q-2$  at time  $t$  is given by,

$$\max_x E_t^{t-q+2} \left[ V_{t+1}^{t-q+2} \exp(-\gamma R x s_{t+1}) \right]. \quad (63)$$

By the calculations in step 1, and using  $\Lambda_t$  as defined in (60), this problem becomes:

$$V^{q-2}(d_{t-K}, \dots, d_t) \quad (64)$$

$$= \max_x E_t^{t-q+2} \left[ -\exp \left( -\frac{1}{2} \left( x_t^{q-1} \right)^2 \gamma^2 ((1 + \beta_0) s_{q-1})^2 - \gamma R x ((1 + \beta_0) d_{t+1} + \Lambda_t) \right) \right]. \quad (65)$$

with  $x_t^{q-1} = d_{t+1} \delta_0 (q-1) + \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t+1-j}$ .

Observe that

$$\begin{aligned} & -\frac{1}{2} \left( d_{t+1} \delta_0 (q-1) + \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t+1-j} \right)^2 \gamma^2 ((1 + \beta_0) s_{q-1})^2 \\ &= -\frac{1}{2} \gamma^2 ((1 + \beta_0) s_{q-1})^2 \left( \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t+1-j} \right)^2 \\ & \quad - \gamma^2 ((1 + \beta_0) s_{q-1})^2 \left( \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t+1-j} \right) \delta_0 (q-1) d_{t+1} \\ & \quad - \frac{1}{2} \gamma^2 ((1 + \beta_0) s_{q-1})^2 (\delta_0 (q-1))^2 d_{t+1}^2, \end{aligned}$$

and that future dividends are the only random variable, with  $d_{t+1} \sim N(\theta_t^{t-q+2}, \sigma^2)$ . Therefore, by Lemma C.3, and with:

$$\begin{aligned} A &= \frac{1}{2} \gamma^2 ((1 + \beta_0) s_{q-1})^2 \left( \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t+1-j} \right)^2 \\ B &= \gamma^2 ((1 + \beta_0) s_{q-1})^2 \left( \delta(q-1) + \sum_{j=1}^K \delta_k(q-1) d_{t+1-j} \right) \delta_0 (q-1) \\ C &= \frac{1}{2} \gamma^2 ((1 + \beta_0) s_{q-1})^2 (\delta_0 (q-1))^2 \end{aligned}$$

we obtain:

$$x_t^{t-(q-2)} = \frac{(1 + \beta_0) s_{q-2}^2 (\sigma^{-2} \theta_t^{t-(q-2)} - B) + \Lambda_t}{R \gamma ((1 + \beta_0) s_{q-2})^2}$$

with  $s_{q-2}^2 \equiv \frac{\sigma^2}{\gamma^2((1+\beta_0)s_{q-1})^2(\delta_0(q-1))^2\sigma^2+1}$ . Therefore,

$$\delta(q-2) = \frac{\alpha(1-R) - s_{q-2}^2(1+\beta_0)\delta_0(q-1)\delta(q-1)\gamma^2((1+\beta_0)s_{q-1})^2}{R\gamma((1+\beta_0)s_{q-2})^2}$$

For  $k \in \{0, \dots, q-1\}$ :

$$\delta_k(q-2) = \frac{(1+\beta_0)s_{q-2}^2(\sigma^{-2}w(k, \lambda, q-2) - [\gamma^2((1+\beta_0)s_{q-1})^2\delta_{k+1}(q-1)\delta_0(q-1)]) + \beta(k)}{R\gamma((1+\beta_0)s_{q-2})^2},$$

$$\delta_k(q-2) = \frac{-(1+\beta_0)s_{q-2}^2[\gamma^2((1+\beta_0)s_{q-1})^2\delta_{k+1}(q-1)\delta_0(q-1)] + \beta(k)}{R\gamma((1+\beta_0)s_{q-2})^2},$$

$$\text{and } \delta_K(q-2) = \frac{\beta(K)}{R\gamma((1+\beta_0)s_{q-2})^2}.$$

By lemma C.1,  $d_{t+1} \sim N(m_t, s_{q-2}^2)$  with  $m_t \equiv -s_{q-2}^2 B + s_{q-2}^2 \sigma^{-2} \theta_t^{t-q+2}$ . Thus, invoking lemma B.1 for this distribution for dividends and  $a = R\gamma(1+\beta_0)$  implies that

$$\begin{aligned} V^{q-2}(d_{t-K}, \dots, d_t) &\asymp -\exp\left(-\frac{1}{2}\left(x_t^{t-(q-2)}\right)^2 (R\gamma)^2((1+\beta_0)s_{q-2})^2\right) \\ &= -\exp\left(-\frac{1}{2}\left(d_t\delta_0(q-2) + \delta(q-2) + \sum_{j=1}^K \delta_k(q-2)d_{t-j}\right)^2 (R\gamma)^2((1+\beta_0)s_{q-2})^2\right) \end{aligned}$$

(the symbol  $\asymp$  means that equality holds up to a positive constant).

STEP 3. We now consider the problem for agents of age  $age \leq q-3$ . Suppose the problem at age  $age+1$  is solved, that is, suppose

$$\begin{aligned} V_{t+1}^{t-age-1} &= V^{age+1}(d_{t+1-K}, \dots, d_{t+1}) \\ &\asymp -\exp\left\{-\frac{1}{2}\left(d_{t+1}\delta_0(age+1) + \delta(age+1) + \sum_{j=1}^K \delta_j(age+1)d_{t+1-j}\right)^2 (R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2\right\}. \end{aligned}$$

The maximization problem is given by:

$$V^{age}(d_{t-K}, \dots, d_t) \equiv \max_x E_t^{t-age} \left[ V_{t+1}^{t-age-1} \exp(-\gamma R^{q-1-age} x((1+\beta_0)d_{t+1} + \Lambda_t)) \right]. \quad (66)$$

By similar calculations to step 2 and Lemma C.3,

$$x_t^{t-age} = \frac{(1+\beta_0)s_{age}^2(\sigma^{-2}\theta_t^{t-age} - B) + \Lambda_t}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}$$

with  $s_{age}^2 \equiv \frac{\sigma^2}{(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2(\delta_0(age+1))^2\sigma^2+1}$ , and

$$B \equiv (R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2 \left( \delta(age+1) + \sum_{j=1}^K \delta_j(age+1)d_{t+1-j} \right) \delta_0(age+1).$$

Therefore

$$\begin{aligned} \delta(age) &= \frac{\alpha(1-R) - s_{age}^2(1+\beta_0)\delta_0(age+1)\delta(age+1)(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}, \\ \delta_k(age) &= \frac{(1+\beta_0)s_{age}^2(\sigma^{-2}w(k, \lambda, age) - [(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2\delta_{k+1}(age+1)\delta_0(age+1)]) + \beta(k)}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2} \\ &\quad k \in \{0, \dots, q-1\}, \\ \delta_k(age) &= \frac{-(1+\beta_0)s_{age}^2[(R^{q-1-(age+1)}\gamma)^2((1+\beta_0)s_{age+1})^2\delta_{k+1}(age+1)\delta_0(age+1)] + \beta(k)}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}, \quad k \in \{q, \dots, K-1\} \\ \delta_K(age) &= \frac{\beta(K)}{R^{q-1-(age)}\gamma((1+\beta_0)s_{age})^2}. \end{aligned}$$

By lemma C.1,  $d_{t+1} \sim N(m_t, s_{age}^2)$  with  $m_t \equiv -s_{age}^2 B + s_{age}^2 \sigma^{-2} \theta_t^{t-q+2}$ . Thus, invoking lemma B.1 for this distribution for dividends and  $a = R^{q-1-age}\gamma(1+\beta_0)$  implies that

$$\begin{aligned} V^{age}(d_{t-K}, \dots, d_t) &\asymp -\exp\left(-\frac{1}{2}\left(x_t^{t-(age)}\right)^2 (R^{q-1-(age)}\gamma)^2((1+\beta_0)s_{age})^2\right) \\ &= -\exp\left(-\frac{1}{2}\left(d_t\delta_0(age) + \delta(age) + \sum_{j=1}^K \delta_k(age)d_{t-j}\right)^2 (R^{q-1-(age)}\gamma)^2((1+\beta_0)s_{age})^2\right). \end{aligned}$$

□

*Proof of Lemma C.5.* Assume it is not the case, i.e.  $1+\beta_0+\beta_1-R\beta_0 \leq 0$ . This implies that  $l(0,1) = (1+\beta_0)w_0 + \beta_1 - R\beta_0 \leq 0$ . From condition (73) we have:

$$0 = \left[ R - \frac{l(1,1)}{(1+\beta_0)} \right] l(0,1) + \frac{l(0,1)^2}{(1+\beta_0)^2} (\beta_1 - R\beta_0) + [1+\beta_0+\beta_1-R\beta_0]$$

Then, since  $\beta_1 - R\beta_0 \leq 0$  by proposition 7.3, for the previous equation to hold it must be that  $\left[ R - \frac{l(1,1)}{(1+\beta_0)} \right] \leq 0$ . However, replacing  $l(1,1)$ , this inequality implies that

$$\left[ R - \frac{(1+\beta_0)(1-w_0) - R\beta_1}{(1+\beta_0)} \right] = \left[ R + \frac{R\beta_1}{1+\beta_0} - (1-w_0) \right] > 0.$$

By Proposition 7.3,  $\frac{R\beta_1}{1+\beta_0} > 0$  and  $R > 1$  by assumption, so we obtained a contradiction. Hence, it must be that  $[1+\beta_0+\beta_1-R\beta_0] > 0$ . □

*Proof of Lemma C.6.* From Lemma B.1, we know that  $x_t^{t-1} = \frac{E_t^{t-1}[s_{t+1}]}{\gamma(1+\beta_0)\sigma^2}$ . Therefore, given our guess for prices and Lemma 7.2, we have:

$$x_t^{t-1} = \frac{E_t^{t-1}[d_{t+1} + p_{t+1} - p_t R]}{\gamma(1+\beta_0)\sigma^2} \quad (67)$$

$$= \frac{(1+\beta_0)\theta_t^{t-1} + \alpha(1-R) + (\beta_1 - R\beta_0)d_t - R\beta_1 d_{t-1}}{\gamma(1+\beta_0)\sigma^2} \quad (68)$$

since  $\theta_t^{t-1} = w_0 d_t + (1-w_0)d_{t-1}$ , we obtain equation (54), where  $l(0,1) = (1+\beta_0)w_0 + \beta_1 - R\beta_0$  and  $l(1,1) = (1+\beta_0)(1-w_0) - R\beta_1$ . We also know from Lemma C.2 that

$$\begin{aligned} V_t^{t-1} &= -\exp\left(-\frac{1}{2} \frac{E_t^{t-q+1}[s_{t+1}]^2}{\gamma(1+\beta_0)\sigma^2}\right) \\ &= -\exp\left(-\frac{1}{2} \frac{(\alpha(1-R) + l(1,1)d_{t-1} + l(0,1)d_t)^2}{\gamma(1+\beta_0)\sigma^2}\right) \\ &= -\exp\left(-\frac{1}{2} \frac{(L_t(1,1) + l(0,1)d_t)^2}{\gamma(1+\beta_0)\sigma^2}\right) \end{aligned}$$

where  $L_t(1,1) \equiv \alpha(1-R) + l(1,1)d_{t-1}$ . Thus, we can write the value function of the generation who is investing for the last time on the market as follows:

$$V_t^{t-1} = -\exp(-A_t - B_t d_t - C d_t^2) \quad (69)$$

where  $A_t \equiv \frac{L_t(1,1)^2}{2\gamma(1+\beta_0)^2\sigma^2}$ ,  $B_t \equiv \frac{L_t(1,1)l(0,1)}{\gamma(1+\beta_0)^2\sigma^2}$ ,  $C \equiv \frac{l(0,1)^2}{2\gamma(1+\beta_0)^2\sigma^2}$ . Using this results to obtain  $V_{t+1}^t$ , the problem of the young generation at time  $t$  is given by:

$$\max_x E_t^t [V_{t+1}^t \exp(-\gamma R x s_{t+1})] \quad (70)$$

From Lemma C.3:

$$x_t^t = \frac{\tilde{\mu}(m, s^2)}{\gamma R \tilde{\sigma}(m, s^2)^2}$$

Where,

$$\begin{aligned} \tilde{\mu}(m, s^2) &= E_{\Phi(m, s^2)}[h(z)] = \alpha(1-R) + (\beta_1 - R\beta_0)d_t - R\beta_1 d_{t-1} + (1+\beta_0)m \\ \tilde{\sigma}(m, s^2)^2 &= V_{\Phi(m, s^2)}[h(d_{t+1})] = (1+\beta_0)^2 s^2 \end{aligned}$$

with  $m = \frac{\theta_t^t - \sigma^2 B_{t+1}}{2C\sigma^2 + 1}$ ,  $s^2 = \frac{\sigma^2}{2C\sigma^2 + 1}$ . Incorporating the fact that  $B_{t+1} = \frac{(\alpha(R-1) + l(1,1)d_t)l(0,1)}{(1+\beta_0)^2\sigma^2}$  and  $\theta_t^t = d_t$  we obtain equation (55) and the respective  $\delta s$ . □

## Appendix OA.2 Proposition 7.2 for the $q = 2$ case

The next lemma specializes the results in Proposition 7.2 for the  $q = 2$  case. It helps illustrate the expressions needed to compute  $\alpha$ ,  $\beta_0$  and  $\beta_1$ .

**Lemma OA.2.1.** *For  $R > 1$  in any linear equilibrium prices are given by:*

$$p_t = \alpha + \beta_0 d_t + \beta_1 d_{t-1} \quad \forall t \in \mathbb{Z} \quad (71)$$

where the coefficients  $\{\alpha, \beta_0, \beta_1\}$  are uniquely determined by the following set of non-linear equations:

$$0 = \alpha(1 - R) \left[ R + \frac{\sigma^2}{s^2} - \frac{l(0, 1)}{1 + \beta_0} \right] - 2R\gamma(1 + \beta_0)^2 \sigma^2 \quad (72)$$

$$0 = l(0, 1) + \frac{1}{R} \frac{\sigma^2}{s^2} (\beta_1 - R\beta_0) + \frac{1}{R} (1 + \beta_0) \left( 1 - \frac{l(1, 1)l(0, 1)}{(1 + \beta_0)^2} \right) \quad (73)$$

$$0 = l(1, 1) - \frac{\sigma^2}{s^2} \beta_1 \quad (74)$$

where  $l(0, 1) \equiv [(1 + \beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0]$  and  $l(1, 1) \equiv [(1 + \beta_0)w(1, \lambda, 0) - R\beta_1]$ .

*Proof of Lemma OA.2.1.* By Proposition 7.1, we have the following demands:

$$x_t^{t-2} = 0 \quad (75)$$

$$x_t^{t-1} = \frac{E_t^{t-1}[s_{t+1}]}{\gamma R(1 + \beta_0) \sigma^2} = \frac{\alpha(1 - R) + l(0, 1) d_t + l(1, 1) d_{t-1}}{\gamma(1 + \beta_0)^2 \sigma^2} \quad (76)$$

$$x_t^t = \frac{E_{\Phi(m, s^2)}[s_{t+1}]}{\gamma R(1 + \beta_0) s^2} = \frac{\alpha(1 - R) + (\beta_1 - R\beta_0) d_t - R\beta_1 d_{t-1} + (1 + \beta_0) m}{\gamma R(1 + \beta_0)^2 s^2} \quad (77)$$

where  $l(0, 1) \equiv (1 + \beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0$ ,  $l(1, 1) \equiv (1 + \beta_0)w(1, \lambda, 0) - R\beta_1$ ,

$$m = \frac{s^2}{\sigma^2} [d_t - \sigma^2 B_{t+1}(1)]$$

$$s^2 = \frac{\sigma^2}{2C(1) \sigma^2 + 1},$$

and

$$B_{t+1}(1) = \frac{\alpha(1 - R) l(0, 1)}{(1 + \beta_0)^2 \sigma^2} + \frac{l(1, 1) l(0, 1)}{(1 + \beta_0)^2 \sigma^2} d_t$$

$$C(1) = \frac{l(0, 1)^2}{(1 + \beta_0)^2 \sigma^2}$$



Therefore:

$$m = \frac{s^2}{\sigma^2} \left[ d_t - \frac{\alpha(1-R)l(0,1)}{(1+\beta_0)^2} - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} d_t \right] = \frac{s^2}{\sigma^2} \left[ -\frac{\alpha(1-R)l(0,1)}{(1+\beta_0)^2} + \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) d_t \right]$$

$$s^2 = \frac{\sigma^2}{2 \frac{l(0,1)^2}{(1+\beta_0)^2 \sigma^2} \sigma^2 + 1} = \frac{(1+\beta_0)^2}{l(0,1)^2 + (1+\beta_0)^2} \sigma^2.$$

Plugging this in the expression for  $x_t^t$ , it follows that

$$x_t^t = \frac{\alpha(1-R) + (\beta_1 - R\beta_0) d_t - R\beta_1 d_{t-1} + (1+\beta_0) \frac{s^2}{\sigma^2} \left[ -\frac{\alpha(1-R)l(0,1)}{(1+\beta_0)^2} + \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) d_t \right]}{\gamma R (1+\beta_0)^2 s^2}$$

$$= \frac{\alpha(1-R) \left[ 1 - \frac{s^2}{\sigma^2} \frac{l(0,1)}{(1+\beta_0)} \right] + \left[ \beta_1 - R\beta_0 + (1+\beta_0) \frac{s^2}{\sigma^2} \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) \right] d_t - R\beta_1 d_{t-1}}{\gamma R (1+\beta_0)^2 s^2}.$$

By Market clearing:

$$1 = \frac{1}{2} \left( \frac{\alpha(1-R) + l(0,1) d_t + l(1,1) d_{t-1}}{\gamma (1+\beta_0)^2 \sigma^2} \right)$$

$$+ \frac{1}{2} \left( \frac{\alpha(1-R) \left[ 1 - \frac{s^2}{\sigma^2} \frac{l(0,1)}{(1+\beta_0)} \right] + \left[ \beta_1 - R\beta_0 + \frac{s^2}{\sigma^2} (1+\beta_0) \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) \right] d_t - R\beta_1 d_{t-1}}{\gamma R (1+\beta_0)^2 s^2} \right)$$

$$= \frac{1}{2} \left( \frac{\alpha(1-R) + l(0,1) d_t + l(1,1) d_{t-1}}{\gamma (1+\beta_0)^2 \sigma^2} \right)$$

$$+ \frac{1}{2} \left( \frac{\alpha(1-R) \frac{\sigma^2}{s^2} \left[ 1 - \frac{s^2}{\sigma^2} \frac{l(0,1)}{(1+\beta_0)} \right] + \left[ \frac{\sigma^2}{s^2} (\beta_1 - R\beta_0) + (1+\beta_0) \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) \right] d_t - \frac{\sigma^2}{s^2} R\beta_1 d_{t-1}}{\gamma R (1+\beta_0)^2 \sigma^2} \right),$$

which implies

$$2\gamma(1+\beta_0)^2 \sigma^2 = (\alpha(1-R) + l(0,1) d_t + l(1,1) d_{t-1})$$

$$+ \frac{1}{R} \left[ \alpha(1-R) \frac{\sigma^2}{s^2} \left[ 1 - \frac{s^2}{\sigma^2} \frac{l(0,1)}{(1+\beta_0)} \right] \right]$$

$$+ \frac{1}{R} \left[ \left[ \frac{\sigma^2}{s^2} (\beta_1 - R\beta_0) + (1+\beta_0) \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) \right] d_t - \frac{\sigma^2}{s^2} R\beta_1 d_{t-1} \right]$$

$$= \alpha(1-R) \frac{1}{R} \left[ R + \frac{\sigma^2}{s^2} - \frac{l(0,1)}{(1+\beta_0)} \right]$$

$$+ \left[ l(0,1) + \frac{1}{R} \frac{\sigma^2}{s^2} (\beta_1 - R\beta_0) + \frac{1}{R} (1+\beta_0) \left( 1 - \frac{l(1,1)l(0,1)}{(1+\beta_0)^2} \right) \right] d_t + \left[ l(1,1) - \frac{\sigma^2}{s^2} \beta_1 \right] d_{t-1}.$$

Therefore  $\{\alpha, \beta_0, \beta_1\}$  solve the following system of equations:

$$0 = \alpha(1 - R) \left[ R + \frac{\sigma^2}{s^2} - \frac{l(0, 1)}{1 + \beta_0} \right] - 2R\gamma(1 + \beta_0)^2 \sigma^2 \quad (78)$$

$$0 = l(0, 1) + \frac{1}{R} \frac{\sigma^2}{s^2} (\beta_1 - R\beta_0) + \frac{1}{R} (1 + \beta_0) \left( 1 - \frac{l(1, 1)l(0, 1)}{(1 + \beta_0)^2} \right) \quad (79)$$

$$0 = l(1, 1) - \frac{\sigma^2}{s^2} \beta_1 \quad (80)$$

where  $l(0, 1) \equiv [(1 + \beta_0)w(0, \lambda, 0) + \beta_1 - R\beta_0]$  and  $l(1, 1) \equiv [(1 + \beta_0)w(1, \lambda, 0) - R\beta_1]$ .  $\square$

### Appendix OA.3 Population Growth

In addition to considering the effects of a one-time shock to population structure, we also explore the implications of population growth.

In this section of the Online Appendix, we consider an OLG model two-period lived agents where the mass of young agents born every period grows at rate  $g$ . For this growth setting, we need to set an initial date for the economy, which we define to be  $t = 0$ . Let  $y_t$  denote the mass of young agents born at time  $t$ ; then  $y_{t+1} = (1 + g)y_t = y_0(1 + g)^t$ . We further denote the total mass of people at any point in time  $t > 0$  as  $n_t$ , and hence  $n_t = y_t + y_{t-1} = (2 + g)y_{t-1}$ . It is easy to check that  $n_t = (1 + g)n_{t-1}$ ; that is, total population grows at rate  $g$ .

The framework is otherwise as in the ‘toy model’ in Section 3 of the main paper. The main difference is that now population is growing over time. As a result, we make a different guess for the price function:

$$p_t = \alpha_0(1 + g)^{-t} + \beta_0 d_t + \beta_1 d_{t-1}$$

We verify this guess using our market clearing condition, which requires the demand of the young and the old to add up to total supply of the asset, one:

$$\begin{aligned} 1 &= y_t \frac{E_t^t [p_{t+1} + d_{t+1}] - Rp_t}{\gamma V [p_{t+1} + d_{t+1}]} + y_{t-1} \frac{E_t^{t-1} [p_{t+1} + d_{t+1}] - Rp_t}{\gamma V [p_{t+1} + d_{t+1}]} \iff \\ 1 &= \frac{y_0(1 + g)^{t-1}}{\gamma(1 + \beta_0)^2 \sigma^2} \left[ (1 + \beta_0) [(1 + g) E_t^t [d_{t+1}] + E_t^{t-1} [d_{t+1}]] + (2 + g) [\alpha_0(1 + g)^{-(t+1)} + \beta_1 d_t - Rp_t] \right] \end{aligned}$$

and after simple algebra,

$$Rp_t = (1 + \beta_0) \left\{ \frac{1 + g}{2 + g} d_t + \frac{1}{2 + g} [(1 - \omega) d_{t-1} + \omega d_t] \right\} + \frac{\alpha_0}{(1 + g)^{t+1}} + \beta_1 d_t - \frac{\gamma \sigma^2 (1 + \beta_0)^2}{y_0 (2 + g) (1 + g)^{t-1}}$$

We plug in  $p_t = \alpha_0(1 + g)^{-t} + \beta_0 d_t + \beta_1 d_{t-1}$  and we use the method of undetermined coeffi-

cients to obtain:

$$\begin{aligned}\alpha_0 &= -\frac{\gamma(1+\beta_0)^2\sigma^2}{R-\frac{1}{1+g}}\frac{(1+g)}{y_0(2+g)} \\ R\beta_0 &= (1+\beta_0)\left(\frac{1+g}{2+g} + \frac{1}{2+g}\omega\right) + \beta_1 \\ R\beta_1 &= (1+\beta_0)\frac{1-\omega}{2+g}\end{aligned}$$

Let  $\alpha_t \equiv \alpha_0(1+g)^{-t}$  and  $\gamma \equiv \frac{y_t}{n_t}$  denote the fraction of young agents, which is easy to verify is constant over time. Then, we can rewrite the above equations as

$$\begin{aligned}\alpha_t &= -\frac{\gamma(1+\beta_0)^2\sigma^2}{R-\frac{1}{1+g}}\frac{1+g}{n_t} \\ R\beta_0 &= (1+\beta_0)(\gamma + (1-\gamma)\omega) + \beta_1 \\ R\beta_1 &= (1+\beta_0)(1-\gamma)(1-\omega).\end{aligned}$$

The latter expressions reveal that the total mass of agents in the market is reflected only in the price constant, while the fraction of young people in the market determines the dividend loadings  $\beta_0$  and  $\beta_1$ . Overall, we see that adding population growth generates to our model generates a positive trend in prices. The relative reliance of prices on the most recent experiences (dividends) is increasing in the population growth rate.